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A stochastic generalized Nash-Cournot model for the northwestern European natural gas markets with a fuel substitution demand function: The S-GaMMES model

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This article presents a stochastic dynamic Generalized Nash-Cournot model to describe the evolution of the natural gas markets. The major gas chain players are depicted including: producers, consumers, storage, and pipeline operators, as well as intermediate local traders. Our economic structure description takes into account market power and the demand representation tries to capture the possible fuel substitution that can be made between the consumption of oil, coal, and natural gas in the overall fossil energy consumption. The demand is made random because of the oil price fluctuations. We take into account the long-term aspects inherent to some markets, in an endogenous way. This particularity of our description makes the model a Generalized Nash Equilibrium problem that needs to be solved using specialized mathematical techniques. The model has been applied to represent the European natural gas market and to forecast, until 2035, after a calibration process, consumption, prices, production, and long-term contract prices and volumes. Finally, we defined and calculated the value of stochastic solution adapted to our model.

Keywords: Energy markets modeling, Game theory, Generalized Nash-Cournot equilibria, Quasi-Variational Inequality, Equilibrium problems, Stochastic programing.

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Abstract

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1 Introduction

When trying to represent the natural gas industry, the modeler may have to deal with uncertainty, at practically all the different gas chain levels. If we consider production, for instance, the exploration activities contain a lot of uncertainty since a producing firm does not know, *a priori*, the amount of gas trapped under the ground before drilling. Regarding the infrastructure, technical hazards may constitute an important uncertainty source in the gas transport. As for the demand, its fluctuations among the months of the year (or the seasonality) is mainly driven by the temperature variation, which is fundamentally a random phenomenon from the point of view of energy economics. Adding to that, uncertainty may be the consequence of political or technical issues that are sometimes hard to take care of in detailed mathematical models. As an example, the Russia/Ukraine dispute over the Russian gas dedicated to Europe that led to an important shortage of supplies, was mainly motivated by political reasons. The unpredictedness of the shortage, which happened twice between 2006 and 2010, may make us consider such situations as random.

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Taking into account randomness in the decisions of a gas industry actor may radically change its planning, as compared with a deterministic foresight's outcome. Indeed, a trader for example who has to choose its gas supplies may want to diversify its sources if he has to deal with security of supply issues. A random demand will deeply influence a producer or a storage operator's investment decisions. Therefore, to be more realistic, it is important to capture randomness of the gas markets when trying to mathematically model them. Nevertheless, though this leads to more realism, considering stochasticity in models is not costless. Indeed, stochastic models are often huge in terms of number of variables and hold computational problems when solving them, which forces the modeler to use decomposition techniques, such as the Benders' decomposition [6], [25] or scenario reduction methods [12]. Therefore, one must select carefully the type of randomness (production, demand, etc.) to consider.

Among all the types of random gas market's characteristics, we decided to model the uncertainty associated with the demand because on the one hand we believe that its impact on the markets' outcome (especially prices and consumption) is the most important and, on the other hand, the demand function specification is the most serious drawback of current gas markets models [51], because it presents an arbitrary aspect in the calibration. The economic literature provides an important panel of numerical models whose objective is to describe the natural gas trade structure while taking into account stochasticity. As an example, we can cite the "Stochastic World Gas Model" (University of Maryland) ([16]), which presents a stochastic extension of the "World Gas Model" ([14]), where the demand is made random. Other interesting works include [24] and [54]. Most of these models consider only randomness of the demand.

A casual look at the oil and gas prices in the spot markets suggests that they are strongly correlated [39]. This is mainly due to two reasons: The first is the long-term contract prices' oil price indexation and the second is related to energy substitution.

Long-term contracts (or LTC) prices between producers and traders have always been indexed by the oil price, to allow natural gas to be a competitive fuel. [1] Since LTC prices constitute a supply marginal cost for the traders, they are correlated to the gas spot prices and, therefore, spot gas and oil prices become correlated too.

Energy substitution also plays an important role in linking the fuels' prices. Indeed, if the consumers are allowed to choose their energy consumption's source, they will go for the cheapest fuel to satisfy their demand (notwithstanding capacity consumption and investment inertia). Therefore, such a consumption feature will ensure all the fuels remain competitive in the market and will link their prices.

Taking into account long-term contract oil price indexation in gas markets modeling requires exogenous data, such as the indexation formula between each pair of producer/trader. Because of a lack of data, we decided to focus mainly on energy substitution to capture the gas and oil prices correlation.

The model we have developed, named S-GaMMES, Stochastic Gas Market Modeling with Energy Substitution is based on an oligopolistic approach to the natural gas markets. The interaction between all the players is a Generalized Nash-Cournot competition and we explicitly take into consideration, in an endogenous way, the long-term contractual aspects (prices and volumes) of the markets that link the producers and the traders. The representation of the demand is new and rich because it includes the possible substitution, within the overall energy consumption,

[1] Currently, some coal prices indexation formulas are being introduced in the contracts.

between different types of fuels. Hence, in our work, we mitigate the market power exerted by the strategic players: they cannot force the natural gas price up freely because some consumers would switch to other fuels to satisfy their demand.

The economic structure we modeled is the one used in the deterministic version of our first model, named GaMMES [1]. In particular, we divide the markets into two stages: the upstream part that represents production and the downstream one, constituted by the different spot markets (end-use consumption markets). Both stages are linked by a set of independent traders. The traders buy gas from the producers on a long-term contract basis and bring it to the spot markets where market power is exerted. Both producers and traders have market power and compete via a Nash-Cournot competition. Long-term contracts, production, transportation, and storage investments are endogenous to the model and this property makes our formulation a Generalized Nash-Cournot game.

The specification of the demand function is the one derived from the system dynamics approach presented in [3]. Besides, in order to capture the oil price's fluctuation and the oil/gas price correlation, we decided to model the oil price as a random variable. This property makes the demand function stochastic.

The remaining parts of this paper are as follows: Section 2.1 gives a general description of the chosen economic structure representation. All the players are presented and are divided into two categories: the strategic and the non-strategic ones. The strategic interaction is also detailed. In Sections 2.3 and 2.4, we explain how we estimate the demand function, introduce stochasticity in the demand representation and construct the scenario tree. Section 2.5 is dedicated to the mathematical representation of the markets: the optimization programs associated with all the strategic and non-strategic players are presented and discussed. We also explain in this part how we make the long-term contract prices and volumes endogenous to the model. A set of theorems and theoretical results inherent to S-GaMMES is provided and discussed in Section 2.6. They principally concern long-term contract prices and volumes characteristics. The last section concludes the paper.

2 The model

2.1 Economic description

The economic structure is similar to the one described in GaMMES. We refer to [1] for more details.

The main advantage of the S-GaMMES model is that it takes into account, in an endogenous way, long-term contracts between the independent traders and the producers. Obviously, this representation is quite realistic for the European natural gas trade since the latter is still dominated by long-term selling/purchase prices and volumes. Another advantage inherent to this description is that the inverse demand function explicitly takes into consideration the possible substitution between consumption of natural gas and the competing fuels.

Market power is exerted by the producers and the independent traders in the spot markets, where the competition is modeled thanks to a Nash-Cournot equilibrium.

Considering the energy substitutions in the natural gas demand mitigates the market power that can be exerted by all the strategic players in the end-use markets. Indeed, this is due to the fact that the consumers have the ability to reduce the natural gas share in their energy mixes if the gas market price is much higher than the substitution fuel's (such as oil and coal) price. Therefore, the producers may not have any considerable incentive to reduce their natural gas production in order to force the price up. This model property allows us to take into account the oil/natural gas prices indexation: the Nash-Cournot interaction will link the natural gas price to the coal and oil prices because of the demand function dependence on these parameters.

Standard stochastic natural gas market models, like [24], [54] and [31] usually consider randomness in the demand. If the demand function is considered linear, which is the case in most of these models, $consumption = a - b \times price$, then the parameter a is usually made stochastic using a discrete probability law. This leads to the construction of a scenario tree that captures the dynamics of the model. Unfortunately most of these models give arbitrary probability laws to the demand levels and do not carry out a realistic calibration process. As an example, the parameter a may follow a Gaussian distribution with an arbitrary mean value and variance. In the Stochastic GaMMES model, randomness is also taken care of by the demand level. Indeed, in order to capture the demand fluctuations and make the model more realistic, we introduced stochasticity in the demand via the fluctuations of the oil price. For that purpose, an econometric study of the oil price is carried out in order to deduce and calibrate the probability law of the oil price's dynamic evolution.

The model also takes advantage of a scenario tree representation where each node represents the intersection of randomness and time. The oil price, at each time-step, is hence a random variable that influences the demand function parameters at each scenario node.

The transport and storage infrastructure is modeled using competitive pipeline and storage operators whose objective is to minimize the operation costs. Regarding the transport, the cost includes transportation, congestion, and investment fees. Regarding the storage, the cost includes capacity reservation, storage, withdrawal, and investment fees.

2.2 Notation

The units chosen for the model are the following: quantities in toe (i.e., Ton Oil Equivalent) or Bcm (i.e., 10^9 cubic meters) and unit prices in \$/toe or \$/cm.

The following table summarizes the notation chosen for the exogenous parameters and the endogenous variables.

Exogenous factors

P	set of producers-dedicated traders
I	set of independent traders
D	set of gas consuming countries in the downstream market (no distinction between the sectors) $D \subset N$
T	time $T = \{0, 1, 2, ..., Num\}$
M	set of seasons. Off-peak (low-consumption) and peak (high-consumption) regimes
F	set of all the gas production fields. $F \subset N$
N	set of the nodes
S	set of the storage sites $S \subset N$
A	set of the arcs (topology)
Ω	set of scenario nodes
Ω_l	set of the tree leaves $\Omega_l \subset \Omega$
Rf_f	field f's total gas resources (endowment)
Kf_f	field f's initial capacity of production, year 0
Lf_f	production node f's maximum increase of the production capacity (in %)
Ic_s	injection marginal cost at storage site s (constant)
Wc_s	withdrawal marginal cost at storage site s (constant)
Rc_s	reservation marginal cost at storage site s (constant)
Ls_s	storage node s's maximum increase of the storage capacity (in %)
Pc_f	production cost function, field f
Tc_a	transport marginal cost through arc a (constant)
Tk_a	pipeline initial capacity through arc a, year 0
Ks_s	initial storage capacity at site s, year 0
Is_s	investment marginal costs in storage (constant)
Ip_f	investment marginal costs in production (constant)
Ik_a	investment marginal costs in pipeline capacity through arc a (constant)
La_a	arc a's maximum increase of the transport capacity (in %)
O	incidence matrix $\in \mathrm{M}_{F\times P}$. $O_{fp} = 1$ if and only if producer p owns field f
B	incidence matrix $\in \mathrm{M}_{I\times D}$. $B_{id} = 1$ if and only if trader i is located at the consumption node d
$M1$	incidence matrix $\in \mathrm{M}_{F\times N}$. $M1_{fn} = 1$ if and only if node n has field f
$M2$	incidence matrix $\in \mathrm{M}_{I\times N}$. $M2_{in} = 1$ if and only if trader i is located at node n
$M3$	incidence matrix $\in \mathrm{M}_{D\times N}$. $M3_{dn} = 1$ if and only if node n has market d
$M4$	incidence matrix $\in \mathrm{M}_{S\times N}$. $M4_{sn} = 1$ if and only if node n has storage site s
$M5$	incidence matrix $\in \mathrm{M}_{A\times N}$. $M5_{an} = 1$ if and only if arc a starts at node n
$M6$	incidence matrix $\in \mathrm{M}_{A\times N}$. $M6_{an} = 1$ if and only if arc a ends at node n
(ω)	probability of occurence of scenario node ω
$t(\omega)$	time associated with scenario node ω
H	maximum value for the quantities produced and consumed
δ^{ω}_{md}	an inverse demand function parameter
β^{ω}_{md}	an inverse demand function parameter
γ^{ω}_{md}	an inverse demand function parameter
pc^{ω}_{md}	an inverse demand function parameter

fl_f	field f's flexibility: the maximum spread between production during off-peak and peak seasons
min_{pi}	percentage of the minimum quantity that has to be exchanged on the long-term contract trade between i and p
δ	discount factor
$delay_{s,i,p}$	period of time necessary to undertake technical investments
$loss_a$	loss factor through arc a
dep_f	depreciation factor of the production capacity at field f

Endogenous variables

x^{ω}_{mfpd}	quantity of gas produced by p from field f for the end-use market d, scenario node ω, season m in Bcm
zp^{ω}_{mfpi}	quantity of gas produced by p from field f dedicated to the long-term contract with trader i, scenario node ω, season m in Bcm
zi^{ω}_{mpi}	quantity of gas bought by trader i from producer p with a long-term contract scenario node ω, season m in Bcm
up_{pi}	quantity of gas sold by producer p to trader i with a long-term contract, each year in Bcm
ui_{pi}	quantity of gas bought by trader i from producer p on the long-term contract, each year in Bcm
y^{ω}_{mid}	quantity of gas sold by i to the market d, scenario node ω, season m in Bcm
ip^{ω}_{fp}	producer p's increase of field f's production capacity, due to investments in production, scenario node ω in Bcm/time unit
q^{ω}_{mfp}	production of producer p from field f, scenario node ω, season m in Bcm
p^{ω}_{md}	market d's gas price, result of the Cournot competition between all the traders, scenario node ω, season m in \$/cm
η_{pi}	long-term contract price contracted between producer p and trader i in \$/cm
r^{ω}_{is}	amount of storage capacity reserved by trader i at site s, scenario node ω in Bcm
in^{ω}_{is}	volume injected by trader i at site s, scenario node ω in Bcm
is^{ω}_{s}	increase of storage capacity at site s, scenario node ω due to the storage operator investments in Bcm/time unit
ik^{ω}_{a}	increase of the pipeline capacity through arc a, scenario node ω, due to the TSO investments in Bcm/time unit
$fp^{\omega}_{m,p,a}$	gas quantity that flows through arc a from producer p scenario node ω, season m in Bcm
$fi^{\omega}_{m,i,a}$	gas quantity that flows through arc a from trader i scenario node ω, season m in Bcm
$\tau^{\omega}_{m,a}$	the dual variable associated with arc a capacity constraint scenario node ω, season m in Bcm/season. It represents the congestion transportation cost over arc a

The previous table is divided into two parts. The upper half represents the exogenous parameters or functions whereas the lower half represents the different decision variables and the inherent retail prices.

The indices p, d, i, f, n, s, a, m, ω and t are such that $p \in P$, $d \in D$, $i \in I$ $f \in F$, $n \in N$, $s \in S$, $a \in A$, $m \in M$, $\omega \in \Omega$ and $t \in T$. In the remainder of the paper and according to the context, a node can either represent a geographical location (of a production field, a consumption market or a storage site) or a location in the scenario tree.

The long-term contract between producer p and trader i fixes both a unit selling price and an amount to be purchased by the independent trader i each year from producer p. Both price and quantity will be specified endogenously by the model.
Matrix O is such that $O_{fp} = 1$ if producer p owns field f and $O_{fp} = 0$ otherwise.

Figure 1 represents a schematic overview of S-GaMMES.

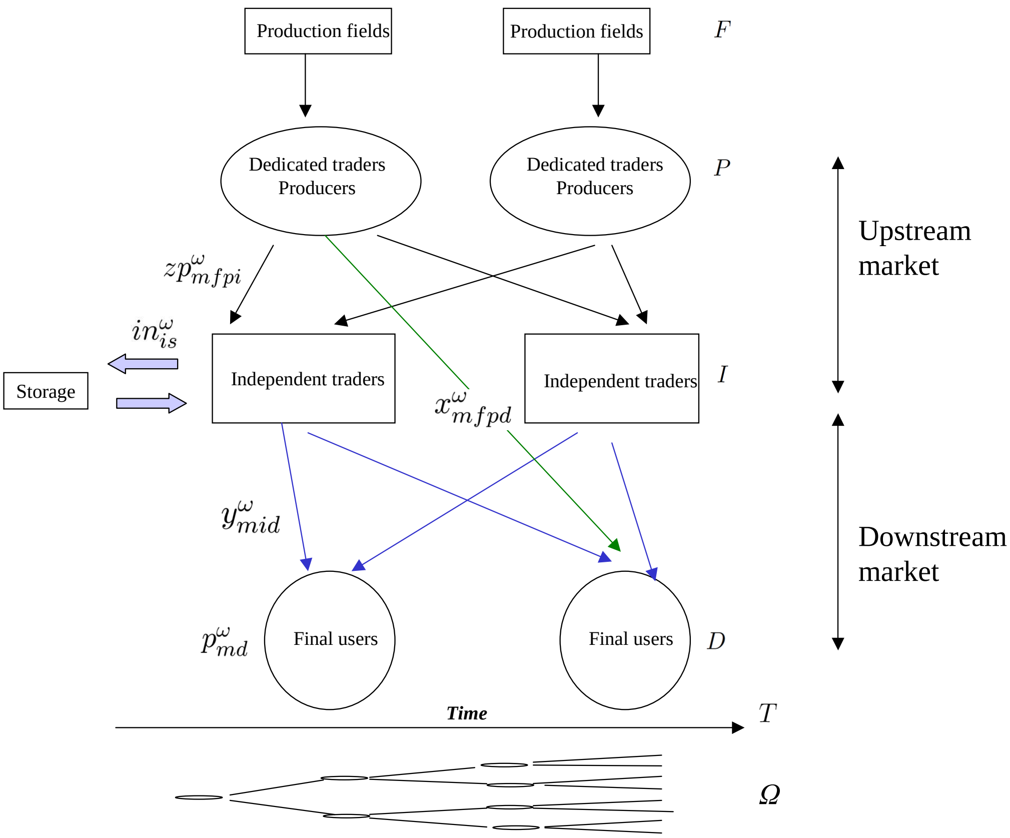


Figure 1:
The market representation in S-GaMMES

2.3 The inverse demand function

We need to specify a functional form for the inverse demand function which links the price p_d at market d to the quantity brought to the market. Most of the natural gas models [49], [48], [41], [14] do not take into account fuel substitution. Let h^{ω}_{md} be the specific inverse demand function in market d, season m of scenario node ω. We assume that the long-term contract quantities do not directly influence the market competition price, which is to say that $p^{\omega}_{md} = h^{\omega}_{md}(\sum_i y^{\omega}_{mid} + \sum_f \sum_p x^{\omega}_{mfpd})$. (Actually, this assumption is necessary to guarantee the concavity of the objective functions of each strategic player's maximization problem, regardless of the quantities decided by the other competitors. Otherwise, this assumption can be dropped if linear functions are used).
As mentioned in the introduction, we want to capture the inter-fuel substitution in the global energy consumption. To be able to do so, we used a system dynamics approach that models the behavior of the consumers who have to decide whether they invest in new burners that use either oil, coal or natural gas. The model is fully developed in [3]. If we denote by Q^{ω}_{md} the quantity $\sum_i y^{\omega}_{mid} + \sum_f \sum_p x^{\omega}_{mfpd}$, the gas demand study [3] provides the following inverse demand function:

$$p^{\omega}_{md} = \begin{array}{ll} pc^{\omega}_{md} + \frac{1}{\gamma^{\omega}_{md}} \text{atanh}\left(\frac{\delta^{\omega}_{md}+\beta^{\omega}_{md}-Q^{\omega}_{md}}{\delta^{\omega}_{md}}\right) & \text{if } Q^{\omega}_{md} \geq \beta^{\omega}_{md} + \frac{\delta^{\omega}_{md}\beta^{\omega}_{md}}{\delta^{\omega}_{md}+\beta^{\omega}_{md}} \\ p'c^{\omega}_{md} + \frac{1}{\gamma'^{\omega}_{md}} \text{atanh}\left(\frac{\delta'^{\omega}_{md}+\beta'^{\omega}_{md}-Q^{\omega}_{md}}{\delta'^{\omega}_{md}}\right) & \text{if } Q^{\omega}_{md} \leq \beta^{\omega}_{md} + \frac{\delta^{\omega}_{md}\beta^{\omega}_{md}}{\delta^{\omega}_{md}+\beta^{\omega}_{md}} \end{array} \quad (1)$$

where the parameters δ, β, γ and pc, which are time- and season-dependent must be calibrated.

The distinction between the domains $Q^{\omega}_{md} \geq \beta^{t}_{md} + \frac{\delta^{\omega}_{md}\beta^{\omega}_{md}}{\delta^{\omega}_{md}+\beta^{\omega}_{md}}$ and $Q^{\omega}_{md} \leq \beta^{\omega}_{md} + \frac{\delta^{\omega}_{md}\beta^{\omega}_{md}}{\delta^{\omega}_{md}+\beta^{\omega}_{md}}$ is needed to take into account the anticipated scrapping of burners and to avoid absurd situations where the price rises toward $+\infty$ (and also to guarantee the concavity of the objective functions). The parameters δ', β', γ' and $p'c$ are calculated to guarantee the continuity of h and its derivative h'. To make the price converge toward 0 when the quantity goes to $+\infty$, we need to force $\beta' = 0$.

The function atanh is such that:

$$\forall x \in (-1, 1) \ \text{atanh}(x) = \frac{1}{2} \ln\left(\frac{1+x}{1-x}\right)$$

To calibrate the demand function for the future, we need to specify a scenario for the global fossil energy demand and the oil and coal market prices. Our system dynamics approach [3] will allow us to understand how the global demand is going to be shared between the consumption of the three fuels and explicitly find the natural gas demand function.

Figure 2 gives the typical shape of the demand function used.

In order to have an algorithm convergence in a reasonable time, the inverse demand function has been linearized in S-GaMMES.

2.4 The scenario tree

This section specifies how the scenario tree is constructed in the model. The demand is made random in order to capture the strong fluctuations of the oil price in Europe. The oil price's dynamic evolution will influence the inverse demand function parameters δ, β, γ and pc. Indeed, if the oil price is high in a certain year, consumers will invest more in natural gas burners (the substitute) and therefore, the future demand for natural gas will rise [2]. On the contrary, a low

[2]This argument holds for a constant evolution of the coal price.

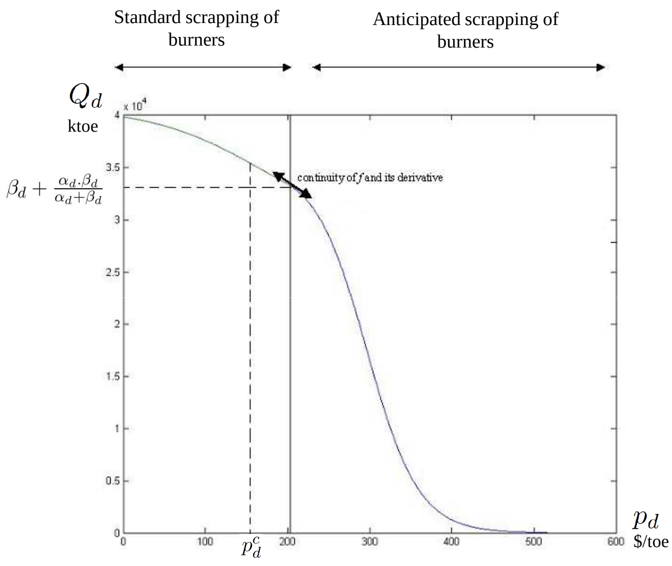


Figure 2:
The demand function

oil price will reduce the future demand for natural gas. The study of the coal price's evolution over time indicates that its fluctuation is negligible compared to the oil one [10]. Therefore, the coal price is not taken as random. To simplify the model, the total gross fossil energy demand is also deterministic.

Let us denote by p_b the chain of the Brent price, with a six-month time-step [3] and ζ_b the corresponding logarithmic percentage price change:

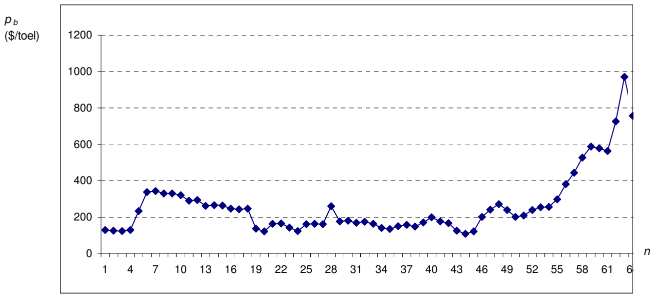
$$\zeta_b = \frac{\ln(p_{b+1}) - \ln(p_b)}{\ln(p_b)} \tag{2}$$

The data base we use for the Brent price is given in [10].
More precisely, p_b is the mean value, over six months, of the Brent price and ζ_b the six-month logarithmic percentage change.

Figure 3 gives the evolution of the price p_b and ζ_b, $b \in \{1, 2, ...64\}$. $b = 1$ corresponds to the period July 1977 to December 1977 and $b = 64$ to the period January 2009 to June 2009.

Figure 4 is a histogram of the variable ζ_b. A visual inspection of the correlogram shows no sign of linear auto-correlation between the variables ζ_b, $b \in \{1, 2, ...64\}$. In addition, the variables' independence has been checked using the BDS test [11] (the BDS statistics with two dimensions 0.008 with probability 0.52). The ζ_b variables can therefore be considered as independent and identically distributed random variables. The Kolmogorov-Smirnov [43] test allows us to state that they have a normal distribution. Indeed, the test did not reject the 0-hypothesis of normality

[3] This time-step is the one that gives the best correlation in our study.



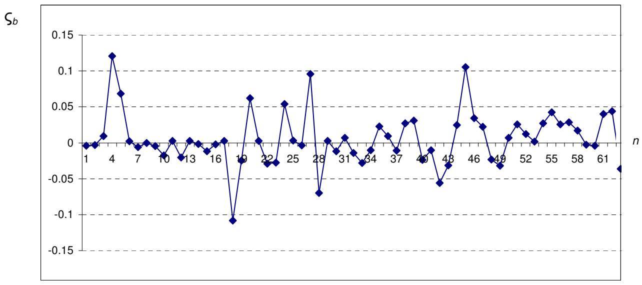


Figure 3:
The evolution of p_b in \$/toe and ζ_b over time.

(Adj. value 1.04 with probability 0.22). The Gaussian fit is provided in Figure 4. This fit is obtained by minimizing the normalized error between a Gaussian distribution and the histogram points of ζ_b. The normalized error is given by the following: if (x_i, y_i), $i \in \{1, 2...n\}$ are the histogram points and $\mathcal{N}_{x0,\sigma}$ a Gaussian distribution, the error $e_{x0,\sigma}$ is:

$$e_{x0,\sigma} = \frac{1}{n} \sum_{i=1}^{n} \left| \frac{y_i - \mathcal{N}_{x0,\sigma}(x_i)}{y_i} \right| \tag{3}$$

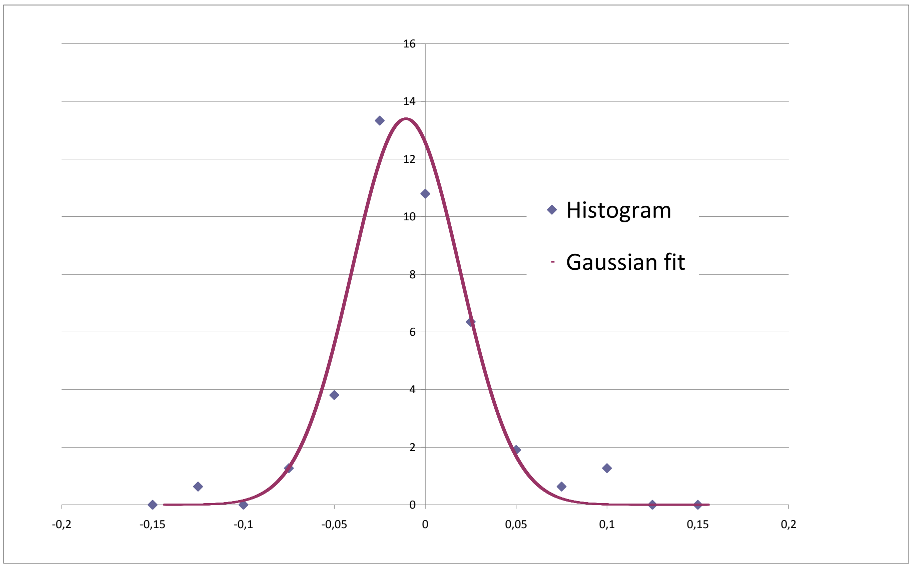


Figure 4:
The histogram of ζ and the Gaussian fit.

The statistical study we carried out provided a normalized error of 0.2, for the Gaussian fit shown in Figure 4. The other numerical results (mean value, variance) will be provided later.

In the representation of the European natural gas trade, we may need to use a time-step longer than six months [4]. Hence, it is worthwhile to explain how we can deduce the new log percentage change's probability density that can be used directly by the model. Let us assume that the model's study time-step is $\kappa\times$ 6 months where $\kappa \in \mathbb{N}$, and call λ the new log percentage change:

$$\lambda_b = \frac{\ln(p_{b+\kappa}) - \ln(p_b)}{\ln(p_b)} \tag{4}$$

λ_b takes into account the $\kappa\times 6$ months offset. In our case, κ=10 (relation between five years and six moinths time-steps). The relationship between λ_b and ζ_b is given using the following lemmas and theorems:

[4] Like in the deterministic version, we typically use a five-year time-step in the stochastic version.

Lemma 1. $\forall b \in \mathbb{N},\ p_{b+\kappa} = p_b^{\Pi_{i=0}^{\kappa-1}(1+\zeta_{b+i})}$

Proof. Theorem 1's proof is straightforward: using equation (2), we can deduce that:

$$\forall b,\ p_{b+1} = p_b^{1+\zeta_b} \tag{5}$$

Hence

$$\begin{aligned}
p_{b+\kappa} &= p_{b+\kappa-1}^{1+\zeta_{b+\kappa-1}} \\
&= p_{b+\kappa-2}^{(1+\zeta_{b+\kappa-2})(1+\zeta_{b+\kappa-1})} \\
&= p_{b+\kappa-3}^{(1+\zeta_{b+\kappa-3})(1+\zeta_{b+\kappa-2})(1+\zeta_{b+\kappa-1})} \\
&= \ldots \\
&= p_b^{\Pi_{i=0}^{\kappa-1}(1+\zeta_{b+i})}
\end{aligned}$$

□

The previous equation can be rewritten as follows:

$$\ln(p_{b+\kappa}) = \Pi_{i=0}^{\kappa-1}(1+\zeta_{b+i})\ \ln(p_b) \tag{6}$$

Figure 4 shows that the random variable ζ is such that $|\zeta| \leq 0.05$ with a more than 90% probability. Hence, we can write that, in first approximation, $\forall b \in \mathbb{N}, \zeta_b << 1$ and

$$\Pi_{i=0}^{\kappa-1}(1+\zeta_{b+i}) \simeq 1 + \sum_{i=0}^{\kappa-1} \zeta_{b+i} \tag{7}$$

The approximation error can be bounded via the following theorem:

Theorem 1. *If we denote by* $\epsilon = \Pi_{i=0}^{\kappa-1}(1+\zeta_{b+i}) - \left(1 + \sum_{i=0}^{\kappa-1} \zeta_{b+i}\right)$ *the error and* ζ_{max} *the maximum absolute value of* ζ_k*:* $\zeta_{max} = Max\{|\zeta_k|,\ k \in \{b, ..., b+\kappa-1\}\}$*, then:*

$$|\epsilon| \leq (1+\zeta_{max})^\kappa - 1 - \kappa\zeta_{max} \tag{8}$$

Theorem 1 allows us to state that this approximation is valid with an error of 10%, with a 90% probability. Its proof is as follows:

Proof. If we develop $\Pi_{i=0}^{\kappa-1}(1+\zeta_{b+i})$, we find:

$$\Pi_{i=0}^{\kappa-1}(1+\zeta_{b+i}) = \sum_{j=0}^{\kappa} \sum_{\substack{(k_1, k_2, ..., k_j) \in \{b, ..., b+\kappa-1\} \\ k_1 < k_2 < ... < k_j}} \zeta_{k_1}\zeta_{k_2}...\zeta_{k_j} \tag{9}$$

In the sum, the term that corresponds to $j = 0$ is 1 and to $j = 1$ is $\left(\sum_{i=0}^{\kappa-1} \zeta_{b+i}\right)$. Therefore, we can write:

$$\sum_{i=0}^{\kappa-1}(1+\zeta_{b+i}) = \left(1 + \sum_{i=0}^{\kappa-1} \zeta_{b+i}\right) + \sum_{j=2}^{\kappa} \sum_{\substack{(k_1, k_2, ..., k_j) \in \{b, ..., b+\kappa-1\} \\ k_1 < k_2 < ... < k_j}} \zeta_{k_1}\zeta_{k_2}...\zeta_{k_j} \tag{10}$$

Therefore, we have:

$$|\epsilon| = \left| \sum_{j=2}^{\kappa} \sum_{\substack{(k_1, k_2, ..., k_j) \in \{b, ..., b+\kappa-1\} \\ k_1 < k_2 < ... < k_j}} \zeta_{k_1}\zeta_{k_2}...\zeta_{k_j} \right| \tag{11}$$

and we can write :

$$
\begin{aligned}
|\epsilon| &\leq \sum_{j=2}^{\kappa} \sum_{\substack{(k_1,k_2,...,k_j) \in \{b,...,b+\kappa-1\} \\ k_1<k_2<...<k_j}} |\zeta_{k_1}||\zeta_{k_2}|...|\zeta_{k_j}| \\
&\leq \sum_{j=2}^{\kappa} \sum_{\substack{(k_1,k_2,...,k_j) \in \{b,...,b+\kappa-1\} \\ k_1<k_2<...<k_j}} \zeta_{max}^j \\
&= \sum_{j=2}^{\kappa} \binom{\kappa}{j} \zeta_{max}^j \\
&= (1+\zeta_{max})^{\kappa} - 1 - \kappa\zeta_{max}
\end{aligned}
$$

The last equality is obtained exploiting the Newton binomial theorem.

□

As said before, Figure 4 shows that the random variable ζ_{max} is such that $\zeta_{max} \leq 0.05$ with a more than 90% probability. Therefore, $|\epsilon| \leq 0.1$ with more than 90% probability.

Using equations (6) and (7), we can deduce that:

$$\lambda_b = \sum_{i=0}^{\kappa-1} \zeta_{b+i} \tag{12}$$

Since we assumed that ζ_b are independent and identically distributed random variables and since we know that they follow the Gaussian distribution $\mathcal{N}_{x_0,\sigma}$, then we can derive that λ_b are also independent and identically distributed and follow a Gaussian probability distribution $\mathcal{N}_{\kappa x_0,\sqrt{\kappa}\sigma}$.

In order to solve the model in a reasonable time, we decided to use only two scenarios for the oil price at each time-step. Therefore, we have to approximate the logarithmic yield λ's Gaussian probability density $\mathcal{N}_{\kappa x_0,\sqrt{\kappa}\sigma}$ by a two-value probability law. Let us call λ_1 and λ_2 the two possible values of the random variable λ that will be used by the model, p and $1-p$ the associated probabilities. The goal now is to find λ_1, λ_2 and p.

The mean value and the standard deviation of λ are respectively κx_0 and $\sqrt{\kappa}\sigma$. Therefore, we can write:

Lemma 2. *λ_1, λ_2 and p verify*

$$
\begin{aligned}
p\lambda_1 + (1-p)\lambda_2 &= x_0 &\text{(13a)} \\
p\lambda^2 + (1-p)\lambda_2^2 - x_0^2 &= \kappa\sigma^2 &\text{(13b)}
\end{aligned}
$$

Proof. Equation (13a) equates the average of the two value probability law $(\lambda_1, \lambda_2, p)$ and the Gaussian distribution. Equation (13b) does the same with the variance. □

Equations (13a) and (13b) allow us to state that (assuming that $p \notin \{0,1\}$):

$$
\begin{aligned}
\lambda_1 &= x_0 + \frac{\sigma}{\sqrt{p}}\sqrt{1-p} &\text{(14a)} \\
\lambda_2 &= x_0 - \frac{\sigma}{\sqrt{p}}\left(\frac{1}{\sqrt{1-p}} - \sqrt{1-p}\right) &\text{(14b)}
\end{aligned}
$$

Since we are looking for three variables $(\lambda_1, \lambda_2, p)$, we need to impose a third equation. In our case, we added the following relation:

$$\lambda_1 = -\lambda_2$$

in order to capture the increasing and decreasing fluctuations of the oil price. A nonnegative value for λ implies an increase of the oil price, whereas a negative value means a decrease of the oil price. In our case, $\lambda_1 \geq 0$ and $\lambda_2 \leq 0$ correspond respectively to an increase and decrease of the Brent price.

The study of the Brent price between 1977 and 2009 gives the follwing values for λ_1, λ_2 and p:

λ_1	0.1
λ_2	-0.1
p	0.15

These values have been calculated for a five-year evolution of the Brent price.
To summarize, the oil price evolves via the following formula:

$$p_{b+\kappa} = p_b^{1+\lambda_1} \quad \text{with probability } p \tag{15a}$$

$$p_{b+\kappa} = p_b^{1+\lambda_2} \quad \text{with probability } 1-p \tag{15b}$$

Relations (15a) and (15b) suggest that the oil price is modeled as a Markov chain. This assumption has been verified and used in some statistical studies of the oil price [53].

To calibrate the demand function for the future, we need to specify a deterministic scenario for the global fossil energy demand and the coal markets' prices. The oil price evolution will create the scenario tree as follows: at each time-step the oil price can follow respectively equation (15a) or (15b) with probability p and $1-p$. In the stochastic version, the model's time scope is 2000-2035, with a time resolution of five years. In order to keep the model solvable in a reasonable time, we considered randomness only for the first five time-steps, until 2025. Starting from 2025, the oil price follows the trend forecast by the European Commission [18]: an increase by 3.7% per year. The corresponding log-change percentage in that case is called μ.

Figure 5 gives a schematic description of the scenario tree for the oil price and therefore for the demand function parameters. There are 31 nodes and seven time-steps (35 years). Node 0, which is the top of the scenario tree corresponds to the 2000-2004 time period. Note that randomness occurs starting from 2010.

Figure 6 gives the values of the different scenario nodes weights $\pi(\omega)$ of the tree.

2.5 The mathematical description

This section details the mathematical description of the model. It presents the optimization problems of all the supply chain players.[5]

Each node of the scenario tree represents the intersection of randomness with time. The first-stage variables are all the ones decided by all the players at node 0 and 1, which are deterministic.

[5]Note that the dual variables are written in parentheses next to their associated constraints.

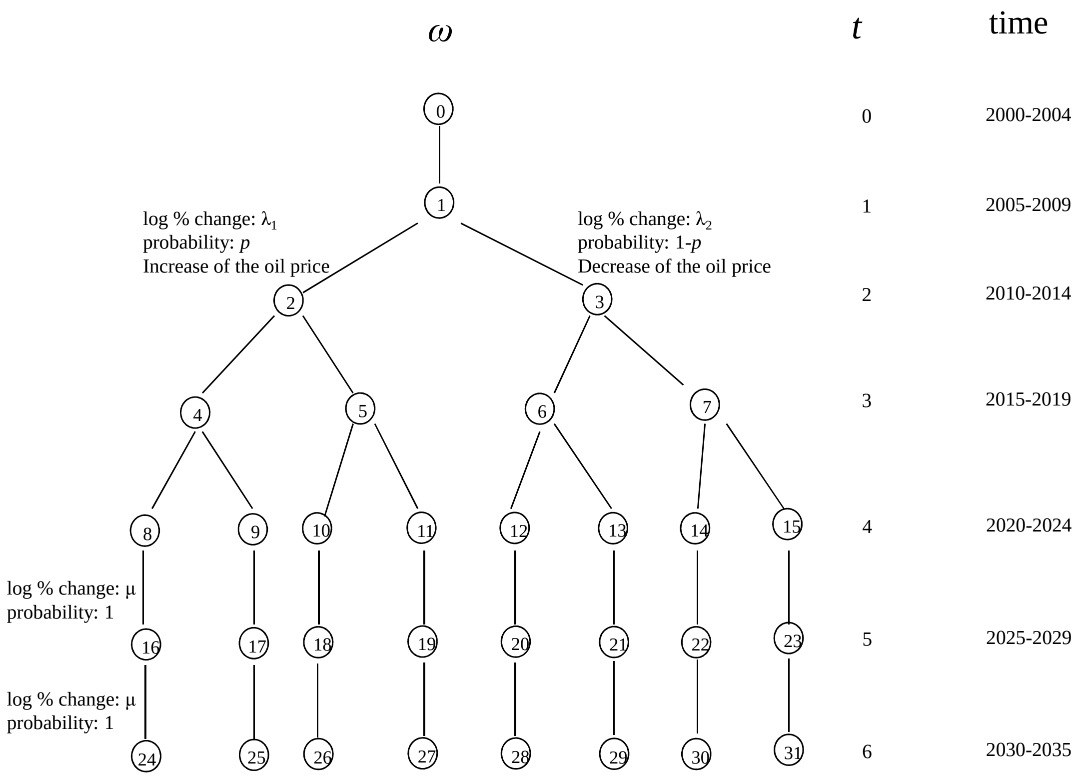


Figure 5:
The scenario tree.

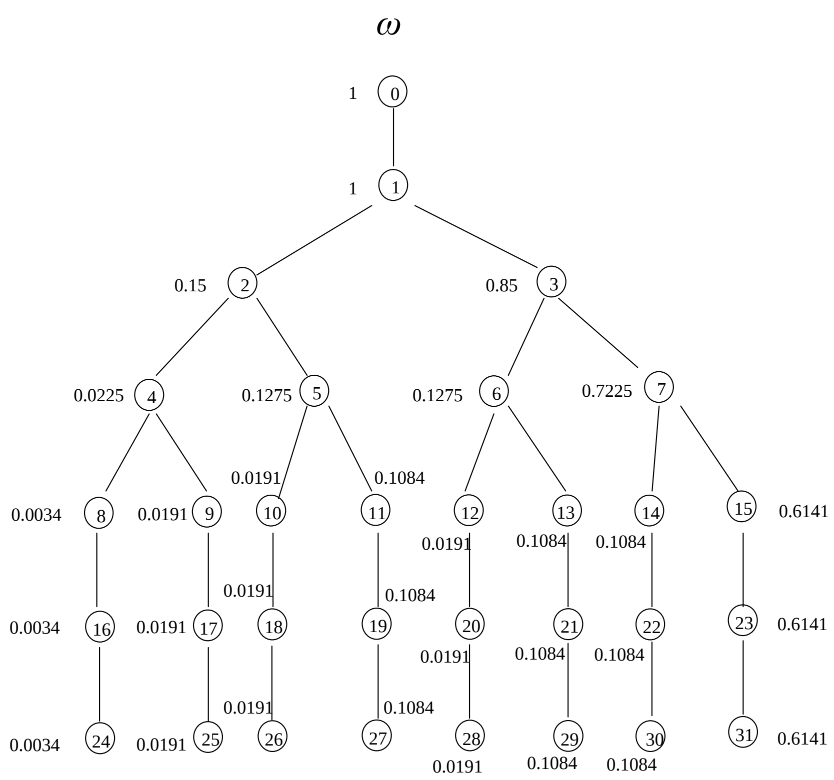


Figure 6:
The scenario tree weights.

Once these variables have been chosen, they cannot be changed later, in the rest of the time periods (or nodes). Similarly, the decisions made at nodes 2 and 3 will influence the market outcome at all the forthcoming nodes $\omega \in \{4, 5, ...31\}$ especially the production, transport, and storage investments. More generally, an investment or a contractual decision made at node ω will remain unchanged and will influence the market structure at all the nodes ω' that follow ω. In the rest of the paper, when two scenario nodes ω and ω' are related, we will write:

$$\omega \leq \omega'$$

if ω' is a successor of ω (or ω is a predecessor of ω'). For example, in the scenario tree, *node* $4 \leq$ *node* 4 and *node* 25.

In order to take into account the different investment delays, we need to consider the strict successors of a particular node. When two scenario nodes ω and ω' are related, we will write:

$$\omega < \omega' \Leftrightarrow \omega \leq \omega' \text{ and } \omega \neq \omega'$$

if ω' is a strict successor of ω (or ω is a strict predecessor of ω'). For example, in the scenario tree, *node* $4 <$ *node* 8 and *node* 25.

Using this scenario tree approach, we do not need to take into account non-anticipativity conditions, because we define a relation between the nodes (successors and predecessors). From the programming perspective, these relations have been included by using incidence matrices $M7$ and $M8$: $M7(\omega, \omega') = 1$ if and only if $\omega \leq \omega'$, otherwise, $M7(\omega, \omega') = 0$ and $M8(\omega, \omega') = 1$ if and only if $\omega < \omega'$, otherwise, $M8(\omega, \omega') = 0$.

Producer p's maximization program is given below. The corresponding decision variables are zp^{ω}_{mfpi}, x^{ω}_{mfpd}, ip^{ω}_{fp}, q^{ω}_{mfp} and up_{pi}.

Max

$$\sum_{\omega,m,f,i} \pi(\omega)\delta^{t(\omega)}(\eta_{pi})(zp^{\omega}_{mfpi})$$
$$+ \sum_{\omega,m,f,d} \pi(\omega)\delta^{t(\omega)} \left(p^{\omega}_{md}(x^{\omega}_{mfpd} + \overline{x^{\omega}_{mfpd}})\right) x^{\omega}_{mfpd}$$
$$- \sum_{\omega,f} \pi(\omega)\delta^{t(\omega)} Pc_f \left(\sum_{\omega' \le \omega} \sum_m q^{\omega'}_{mfp}, Rf_f\right)$$
$$+ \sum_{\omega,f} \pi(\omega)\delta^{t(\omega)} Pc_f \left(\sum_{\omega' < \omega} \sum_m q^{\omega'}_{mfp}, Rf_f\right)$$
$$- \sum_{\omega,f} \pi(\omega)\delta^{t(\omega)} Ip_f(ip^{\omega}_{fp})$$
$$- \sum_{\omega,m,a} \pi(\omega)\delta^{t(\omega)}((Tc_a + \tau^{\omega}_{m,a})fp^{\omega}_{m,p,a})$$

such that:

$$\forall \omega,\ f, \qquad \sum_p \sum_{\omega' \le \omega} \sum_m q^{\omega}_{mfp} - Rf_f \le 0 \qquad (\phi^{\omega}_f) \tag{16a}$$

$$\forall \omega,\ f,\ m, \qquad \sum_p q^{\omega}_{mfp} - Kf_f(1-dep_f)^{t(\omega)} - \sum_p \sum_{\omega' < \omega} ip^{\omega'}_{fp}(1-dep_f)^{t(\omega)-t(\omega')} \le 0 \qquad (\chi^{\omega}_{mf}) \tag{16b}$$

$$\forall \omega,\ m,\ f, \qquad -q^{\omega}_{mfp} + \left(\sum_i zp^{\omega}_{mfpi} + \sum_d x^{\omega}_{mfpd}\right) \le 0 \qquad (\gamma^{\omega}_{mfp}) \tag{16c}$$

$$\forall \omega,\ f,\ p, \qquad \sum_m((-1)^m q^{\omega}_{mfp}) - fl_f \le 0 \qquad (\vartheta 1^{\omega}_{fp}) \tag{16d}$$

$$\forall \omega,\ f,\ p, \qquad -\sum_m((-1)^m q^{\omega}_{mfp}) - fl_f \le 0 \qquad (\vartheta 2^{\omega}_{fp}) \tag{16e}$$

$$\forall \omega,\ f,\ d,\ m, \qquad x^{\omega}_{mfpd} - O_{fp}H \le 0 \qquad (\epsilon 1^{\omega}_{mfpd}) \tag{16f}$$

$$\forall \omega,\ f,\ i,\ m, \qquad zp^{\omega}_{mfpi} - O_{fp}H \le 0 \qquad (\epsilon 2^{\omega}_{mfpi}) \tag{16g}$$

$$\forall \omega,\ f,\ m, \qquad q^{\omega}_{mfp} - O_{fp}H \le 0 \qquad (\epsilon 3^{\omega}_{mfp}) \tag{16h}$$

$$\forall \omega,\ f, \qquad ip^{\omega}_{fp} - O_{fp}H \le 0 \qquad (\epsilon 4^{\omega}_{fp}) \tag{16i}$$

$$\forall \omega,\ f, \qquad \sum_p ip^{\omega}_{fp} - Lf_f Kf_f(1-dep_f)^{t(\omega)} - Lf_f \sum_p \sum_{\omega' < \omega} ip^{\omega'}_{fp}(1-dep_f)^{t(\omega)-t(\omega')} \le 0 \qquad (\iota p^{\omega}_f) \tag{16j}$$

$$\forall \omega,\ m,\ n, \qquad \sum_a M6_{an} fp^{\omega}_{m,p,a}(1-loss_a) - \sum_a M5_{an} fp^{\omega}_{m,p,a} + \sum_f q^{\omega}_{mpf} M1_{fn} - \sum_d \sum_f x^{\omega}_{mfpd} M3_{dn} - \sum_i \sum_f zp^{\omega}_{mfpi} M2_{in} = 0 \qquad (\alpha p^{\omega}_{m,p,n}) \tag{16k}$$

(16l)

$$
\begin{array}{llll}
\forall \omega,\ i, & up_{pi} - \sum_{f,m} zp^{\omega}_{mfpi} & = 0 & (\eta p^{\omega}_{pi}) \quad (17a) \\
\forall\ p,\ i, & ui_{pi} - up_{pi} & = 0 & (\eta_{pi}) \quad (17b) \\
\forall \omega,\ m,\ d,\ i,\ f, & zp^{\omega}_{mfpi},\ x^{\omega}_{mfpd},\ ip^{\omega}_{fp},\ q^{\omega}_{mfp},\ up_{pi} & \geq 0 &
\end{array}
$$

We denote by $\overline{x^{\omega}_{mfpd}}$ the total amount of gas brought at node ω, season m to the market d by all the players different from producer p.

The term

$$\sum_{\omega,m,f,i} \pi(\omega)\delta^{t(\omega)}(\eta_{pi})(zp^{\omega}_{mfpi}) + \sum_{\omega,m,f,d} \pi(\omega)\delta^{t(\omega)} \left(p^{\omega}_{md}(x^{\omega}_{mfpd} + \overline{x^{\omega}_{mfpd}})\right) x^{\omega}_{mfpd}$$

is the revenue, which is obtained from the sales from the long-term contract sales to the independent traders or directly from the retail markets.
The term

$$\sum_{\omega,f} \pi(\omega)\delta^{t(\omega)} Pc_f \left(\sum_{\omega' \leq \omega} \sum_{m} q^{\omega'}_{mfp}, Rf_f \right) - \sum_{\omega,f} \pi(\omega)\delta^{t(\omega)} Pc_f \left(\sum_{\omega' < \omega} \sum_{m} q^{\omega'}_{mfp}, Rf_f \right)$$

is the actualized production cost. This term's explanation is as follows:
The production cost (at field f) Pc_f depends on two variables, the total quantity produced, which will be denoted q and the natural gas resources Rf_f. The Golombek production cost function we used is as follows:

$$\forall q \in [0, Rf_f),\ Pc_f(q, Rf_f) = a_f q + b_f \frac{q^2}{2} - \ Rf_f c_f \left(\frac{Rf_f - q}{Rf_f} \ln \left(\frac{Rf_f - q}{Rf_f} \right) + \frac{q}{Rf_f} \right) \tag{18}$$

or if written for the marginal production cost

$$\forall q \in [0, Rf_f),\ \frac{\mathrm{d}Pc_f}{\mathrm{d}q} = a_f + b_f q \ + \ c_f \ln \left(\frac{Rf_f - q}{Rf_f} \right) \tag{19}$$

In our model, the production cost function is dynamic. The gas volume available to be extracted is dynamically reduced at each period, taking into account the exhaustivity of the resource.
If at time-step 1, the production is $q1$ and at time-step 2 $q2$, the total cost is hence:

$$cost = Pc_f(q1, RES_f) + \delta(Pc_f(q1 + q2, RES_f) - Pc_f(q1, RES_f))$$

Thus, to estimate the cost at scenario node ω, we need to calculate the production cost of the sum over all the extracted volumes until node ω and subtract the cost we have cummulated at all the strict predecessor nodes to ω.

The term

$$\sum_{\omega,f} \pi(\omega)\delta^{t(\omega)} Ip_f(ip^{\omega}_{fp})$$

is the investment cost in production at the different production fields.

The term

$$\sum_{\omega,m,a} \pi(\omega)\delta^{t(\omega)}((Tc_a + \tau^{\omega}_{m,a})fp^{\omega}_{m,p,a})$$

is the transport and congestion costs charged by the pipeline operator to producer p. The dual variable $\tau^{\omega}_{m,ar}$ is associated with the pipeline capacity constraint through the arc a. It represents the congestion price on the corresponding pipeline (see the transport operator optimization problem for a more detailed explanation).

The explanation of the constraints is straightforward:
The constraint (16a) bounds each field's production by its reserves.
The constraint (16b) bounds the seasonal quantities produced by each field's production capacity, taking explicitly into account the different dynamic investments, that decrease with time because of the production depreciation factor. To take into consideration the investment delays, we account only for the invested capacities at the strict predecessor nodes. This corresponds to a five-year investment delay (the time-step of the model).
The constraint (16c) states that the total production must be greater than the sales (to the long-term and spot markets). The constraints (16d) and (16e) can be rewritten as follows:

$$\forall \omega,\ f,\ p,\ |\sum_m((-1)^m q^{\omega}_{mfp})| \leq fl_f$$

This fixes a maximum spread between the off-peak/peak production at each field. $(-1)^m$ is equal to 1 in the off-peak season and -1 in the peak season.
The constraint (16k) is a market-clearing condition at each node, regarding the flows from producer p depending on whether this node is a production field, an independent trader location or a demand market.
The constraint (16j) bounds the capacity expansion of each production node f: each year, the investment decided to increase the production capacity is less than $100 \times Lf_f$ percent the installed capacity at that year. A historical study of the capacity expansion of some production nodes allowed us to calibrate the value of Lf_f: $Lf_f = 0.20$.
The constraint (17a) equates the sales of producer p for the long-term contracts to the contracted volume up_{pi}, each scenario node.
The constraint (17b) describes the following: For each pair of producer/independent trader (p, i), the gas quantity sold by p in the long-term contract market must be equal to the gas quantity purchased by i. Therefore, this is a supply/demand equation in the long-term contracts market. The associated dual variable η_{pi} is the corresponding contract unit selling/purchase price, because we do not assume the existence of market power in the long-term contract trade. Using this technique, it is possible to make the long-term contract prices and volumes endogenous to the description so that they become an output of the model.
The constraint (and the similar other ones) (16f) allows producer p to use only the fields he owns (for production, investments, sales, etc.). We recall that the incidence matrix O is such as $O_{fp} = 1$ if and only if producer p owns field f, otherwise, $O_{fp} = 0$.

Independent trader i's maximization program is given below. The corresponding decision variables are zi^{ω}_{mpi}, y^{ω}_{mid}, r^{ω}_{is}, in^{ω}_{is} and ui_{pi}.

Max

$$\sum_{\omega,m,d} \pi(\omega)\delta^{t(\omega)} \left(p^{\omega}_{md}(y^{\omega}_{mid} + \overline{y^{\omega}_{mid}})y^{\omega}_{mid}\right)$$
$$- \sum_{\omega,p,m} \pi(\omega)\delta^{t(\omega)} \left(\eta_{pi} zi^{\omega}_{mpi}\right)$$
$$- \sum_{\omega,s} \pi(\omega)\delta^{t(\omega)} \left(Rc_s(r^{\omega}_{is})\right)$$
$$- \sum_{\omega,s} \pi(\omega)\delta^{t(\omega)} \left((Ic_s + Wc_s) in^{\omega}_{is}\right)$$
$$- \sum_{\omega,m,a} \pi(\omega)\delta^{t(\omega)} \left(Tc_a + \tau^{\omega}_{m,a}\right) fi^{\omega}_{m,i,a}$$

such that:

$$\forall \omega,\ m, \quad \sum_p zi^{\omega}_{mfpi} - \left(\sum_d y^{\omega}_{mid} + (-1)^m \sum_s in^{\omega}_{is}\right) = 0 \quad (\psi^{\omega}_{mi}) \tag{20a}$$

$$\forall \omega,\ s, \quad in^{\omega}_{is} - r^{\omega}_{is} \leq 0 \quad (\mu^{\omega}_{is}) \tag{20b}$$

$$\begin{aligned} \forall \omega,\ m,\ n, \quad & \sum_a M6_{an} fi^{\omega}_{m,i,a}(1 - loss_a) - \sum_a M5_{an} fi^{\omega}_{m,i,a} \\ & - \sum_d y^{\omega}_{mfid} M3_{dn} + \sum_p zi^{\omega}_{mpi} M2_{in} \\ & - (-1)^m \sum_s M4_{sn} \left(\sum_i ij^{\omega}_{is}\right) = 0 \quad (\alpha i^{\omega}_{m,p,n}) \end{aligned} \tag{20c}$$

$$\forall \omega,\ p, \quad ui_{pi} - \sum_m zi^{\omega}_{mpi} = 0 \quad (\eta i^{\omega}_{pi}) \tag{20d}$$

$$\forall\ p,\ i, \quad ui_{pi} - up_{pi} = 0 \quad (\eta_{pi}) \tag{20e}$$

$$\forall \omega,\ m,\ p,\ i, \quad - zi^{\omega}_{mpi} + min_{pi} \sum_m zi^{\omega}_{mpi} \leq 0 \quad (\upsilon^{\omega}_{mpi}) \tag{20f}$$

$$\forall \omega,\ m,\ s,\ d, \quad zi^{\omega}_{mpi},\ y^{\omega}_{mid},\ r^{\omega}_{is},\ in^{\omega}_{is},\ ui_{pi} \geq 0$$

The term

$$\sum_{\omega,m,d} \pi(\omega)\delta^{t(\omega)} \left(p^{\omega}_{md}(y^{\omega}_{mid} + \overline{y^{\omega}_{mid}})y^{\omega}_{mid}\right) - \sum_{\omega,p,m} \pi(\omega)\delta^{t(\omega)} \left(\eta_{pi} zi^{\omega}_{mpi}\right)$$

is the net profit.
The term

$$\sum_{\omega,s} \pi(\omega)\delta^{t(\omega)} \left(Rc_s(r^{\omega}_{is})\right)$$

is the storage capacity reservation cost.
The term

$$\sum_{\omega,s} \pi(\omega)\delta^{t(\omega)} \left((Ic_s + Wc_s) in^{\omega}_{is}\right)$$

are the storage/withdrawal costs. [6]

[6] There are no storage losses in the model. They can easily be taken into account by increasing the tansportation losses of the arcs that start at the storage nodes.

The term

$$\sum_{\omega,m,a} \pi(\omega)\delta^{t(\omega)} \left(Tc_a + \tau^{\omega}_{m,a}\right) fi^{\omega}_{m,i,a}$$

is the transport and congestion costs charged by the pipeline operator to the independent trader i.

As for the feasibility set, it is also easy to specify :

The constraint (20a) is a gas quantity balance for each trader. The term $(-1)^m$ is equal to 1 in the off-peak season and -1 otherwise. An implicit assumption we use in the description is that all the storage sites must be "empty" (regardless of the working gas quantitities) at the end of each year.
The equation (20b) implies that each independent trader has to pay for a storage reservation quantity, each year and at each storage site s, to be able to store his gas.
The constraint (20d) forces each trader to purchase the same quantity, in long-term contracts, from each producer and scenario node.
The constraint (20e) is similar to the constraint (17b) of the producers' optimization program. For each pair of producer/independent trader (p, i), the gas quantity sold by p in the long-term contract market must be equal to the gas quantity purchased by i. Therefore, this is a supply/demand equation in the long-term contracts market. The associated dual variable η_{pi} is the corresponding contract unit selling/purchase price, because we do not assume the existence of market power in the long-term contract trade. Using this technique, it is possible to make the long-term contract prices and volumes endogenous to the description so that they become an output of the model.
The constraint (20f) fixes a minimum percentage of the contracted volume, per time unit, min_{pi} that has to be exchanged between p and i each season of each scenario node. Obviously, this constraint is expected to be more saturated in the summer when there is little need for the traders to have an important amount of gas supply.

On the transportation side of our model, we will assume that the producers pay the transport costs to bring natural gas from the production fields to the independent traders' locations and the end-use markets. The traders support the transport costs to store/withdraw gas or bring it to the end-users for their sales. All the distribution costs are implicitly included in the transportation costs we use.

The pipeline operator optimization (cost minimization) program is given below. The corresponding decision variables are $fp^{\omega}_{m,p,a}$, $fi^{\omega}_{m,i,a}$ and ik^{ω}_{a}.

Min

$$\sum_{\omega,m,a} \pi(\omega)\delta^{t(\omega)} \left(Tc_a + \tau^{\omega}_{m,a}\right) \sum_{p} fp^{\omega}_{m,p,a}$$
$$+ \sum_{\omega,m,a} \pi(\omega)\delta^{t(\omega)} \left(Tc_a + \tau^{\omega}_{m,a}\right) \sum_{i} fi^{\omega}_{m,i,a}$$
$$+ \sum_{\omega,a} \pi(\omega)\delta^{t(\omega)} Ik_a(ik^{\omega}_a)$$

such that:

$$\forall \omega,\ m,\ a, \qquad \sum_p fp^{\omega}_{m,p,a} + \sum_i fi^{\omega}_{m,i,a} - \left(Tk_a + \sum_{\omega' < \omega} ik^{\omega'}_a\right) \leq 0 \qquad (\tau^{\omega}_{m,a}) \tag{21a}$$

$$\forall \omega,\ a, \qquad ik^{\omega}_a - La_a \left(Tk_a + La_a \sum_{\omega' < \omega} ik^{\omega'}_a\right) \leq 0 \qquad (\iota a^{\omega}_a) \tag{21b}$$

$$\begin{aligned} \forall \omega,\ m,\ p,\ n, \qquad & \sum_a M6_{an} fp^{\omega}_{m,p,a}(1 - loss_a) - \sum_a M5_{an} fp^{\omega}_{m,p,a} \\ & + \sum_f q^{\omega}_{mpf} M1_{fn} - \sum_d \sum_f x^{\omega}_{mfpd} M3_{dn} \\ & - \sum_i \sum_f zp^{\omega}_{mfpi} M2_{in} = 0 \qquad (\alpha p^{\omega}_{m,p,n}) \end{aligned} \tag{21c}$$

$$\begin{aligned} \forall \omega,\ m,\ i,\ n, \qquad & \sum_a M6_{an} fi^{\omega}_{m,i,a}(1 - loss_a) - \sum_a M5_{an} fi^{\omega}_{m,i,a} \\ & - \sum_d y^{\omega}_{mfid} M3_{dn} + \sum_p zi^{\omega}_{mpi} M2_{in} \\ & - (-1)^m \sum_s M4_{sn} \left(\sum_i ij^{\omega}_{is}\right) = 0 \qquad (\alpha i^{\omega}_{m,p,n}) \end{aligned} \tag{21d}$$

$$\forall \omega,\ m,\ a,\ p,\ i, \qquad fp^{\omega}_{m,p,a},\ fi^{\omega}_{m,i,a},\ ik^{\omega}_a \geq 0$$

The objective function contains both the transport/congestion and invesment costs.
The congestion cost through arc a, $\tau^{\omega}_{m,a}$, is the dual variable associated with the constraint (21a). This constraint concerns the physical seasonal capacity of arc a, including the possible node-dependent investments.
The constraint (21b) bounds the capacity expansion of each arc a: each year, the investment decided to increase the transport capacity is less than $100 \times La_a$ percent the installed capacity at that year. In GaMMES, we used the value $La_a = 0.2$.
The other constraints are market-clearing conditions at each node, depending on whether this node is a production field, an independent trader location, a demand market or a storage site, and depending on whether the transportation costs are supported by the producers or the independent traders.
We consider both pipeline and LNG routes for transport. The liquefaction and regasification costs are included in the transportation costs on the LNG arcs. We assume, in the representation that the physical losses occur at the end nodes of the arcs.

The storage operator optimization (cost minimization) program is given below. The corresponding decision variable is is_s^ω.

Min

$$\sum_{\omega,s} \pi(\omega)\delta^{t(\omega)} Is_s(is_s^\omega) + \sum_{i,\omega,s} \pi(\omega)\delta^{t(\omega)} \left((Ic_s + Wc_s)in_{is}^\omega + Rc_s r_{is}^\omega\right)$$

such that:

$$\forall \omega,\ s, \quad \sum_i r_{is}^\omega - Ks_s - \sum_{\omega' < \omega} is_s^{\omega'} \leq 0 \quad (\beta s_s^\omega) \tag{22a}$$

$$\forall \omega,\ s, \quad is_s^\omega - Ls_s Ks_s - Ls_s \sum_{\omega' < \omega} is_s^{\omega'} \leq 0 \quad (\iota s_s^\omega) \tag{22b}$$

$$\forall \omega,\ s, \quad is_s^\omega \geq 0$$

The storage operator only controls the different investments that dynamically increase the storage capacity of each storage node. The incentive this player has to invest is due to the constraint he must satisfy: the capacity available at each storage site must be sufficient to meet the volumes the independent traders have to store each year in the off-peak season. Capacity expansion is bounded and we used the value $Ls_s = 0.2$.

If we take a closer look at the optimization program of a producer, we will notice that his feasibility set depends on the decision variables of the independent traders. Also, the feasibility set of any independent trader's optimization program depends on the producers' decision variables. The situation is similar for the pipeline and storage operators. This particularity makes our formulation (the KKT conditions) a **Generalized Nash-Cournot problem**. Similarly, the Generalized Nash-Cournot problem can also be formulated as a Quasi Variational Inequality problem (QVI). In order to solve the problem, we look for the particular solution that makes the problem a VI formulation [29].

When the KKT conditions are written, we obtain the Mixed Complementarity Problem given in Appendix 2.

2.6 Theoretical results

We refer to Appendix 2 for the MCP formulation of S-GaMMES. This section uses the appendix's equations numbers.

One of the S-GaMMES model's key features is that it cpatures the markets' long-term aspects in an endogenous way, for both long-term contract prices and volumes. In the deterministic version of GaMMES, it can be proved that long-term contracts' prices, or LTC prices, are smaller than the spot market prices.[7] Indeed, since long-term contracts are the only means for the independent traders to obtain gas, LTC prices are to be considered as supply costs for them. Besides, they make a profit by selling natural gas directly to the consumers, in the spot markets. Therefore, if the traders have an incentive to sell gas to the consumers, their revenue must be greater than

[7] Though this situation is less realistic nowadays, given the current high, long-term contract gas prices that are indexed on the oil price.

their costs and consequently, spot prices should be, on average, greater than LTC prices.

These conclusions still hold for the stochastic version of GaMMES. They are explained in the following theorems.

First we prove that our representation of the long-term contracts leads to nonnegative LTC prices.

Theorem 2. *If producer p and trader i contract on the long-term, then the long-term contract price η_{pi} is such as $\eta_{pi} \geq 0$*

Proof. We assume that producer p contracts on the long-term with trader i. This means that the LTC volume up_{pi} is such that $up_{pi} > 0$. Let us denote by d the market where i is located, i.e., d is the only market such as $B_{id} = 1$. Hence, we can write that

$$\forall n \in N,\ M2_{in} = M3_{dn}$$

.

We already know from equation (49i) that:

$$\forall \omega,\ up_{pi} - \sum_{f,m} zp^{\omega}_{mfpi} = 0$$

Hence we can deduce that:

$$\forall \omega,\ \exists f(\omega) \in F \text{ and } m(\omega) \in M \text{ such as } zp^{\omega}_{m(\omega)f(\omega)pi} > 0$$

where we denote by $f(\omega)$ the particular field that producer p may use, at scenario node ω to respect the LTC volume he has to sell to i in season $m(\omega)$.

Because of relation (51e), we can deduce that

$$\forall \omega,\ \exists f(\omega) \in F \text{ such as } \forall m \in M,\ zp^{\omega}_{mf(\omega)pi} > 0$$

Using the complementarity condition of equation (48a), we can deduce that: $\forall \omega,\ \exists f(\omega) \in F$ such as $\forall m \in M$,

$$\pi(\omega)\delta^{t(\omega)}\eta_{pi} - \gamma^{\omega}_{mf(\omega)p} - \epsilon 2^{\omega}_{mf(\omega)pi} - \eta p^{\omega}_{pi} - \sum_{n} M2_{in}\alpha p^{\omega}_{m,p,n} = 0 \qquad (23)$$

Since $zp^{\omega}_{mf(\omega)pi} > 0$, producer p owns the particular field $f(\omega)$ and constraint (16g) is not saturated. Therefore, $\epsilon 2^{\omega}_{mf(\omega)pi} = 0$ and

$$\pi(\omega)\delta^{t(\omega)}\eta_{pi} - \gamma^{\omega}_{mf(\omega)p} - \eta p^{\omega}_{pi} - \sum_{n} M2_{in}\alpha p^{\omega}_{m,p,n} = 0 \qquad (24)$$

To simplify the notation we will denote the term

$$\left(p^{\omega}_{md}(x^{\omega}_{mf(\omega)pd} + \overline{x^{\omega}_{mf(\omega)pd}}) + \frac{\partial p^{\omega}_{md}}{\partial x^{\omega}_{mfpd}}(x^{\omega}_{mf(\omega)pd} + \overline{x^{\omega}_{mf(\omega)pd}}) x^{\omega}_{mf(\omega)pd} \right)$$

by

$$\left(p^{\omega}_{md} + \frac{\partial p^{\omega}_{md}}{\partial x^{\omega}_{mfpd}} x^{\omega}_{mf(\omega)pd} \right)$$

Using relation (48b) we have:

$$\pi(\omega)\delta^{t(\omega)}\left(p^{\omega}_{md}+\frac{\partial p^{\omega}_{md}}{\partial x^{\omega}_{mfpd}}x^{\omega}_{mf(\omega)pd}\right)-\gamma^{\omega}_{mf(\omega)p}-\epsilon 1^{\omega}_{mf(\omega)pd}-\sum_{n}M3_{dn}\alpha p^{\omega}_{m,p,n}\leq 0 \tag{25}$$

Since producer p owns the particular field $f(\omega)$, constraint $(16f)$ is not saturated. Therefore, $\epsilon 1^{\omega}_{mf(\omega)pd}=0$ and

$$\pi(\omega)\delta^{t(\omega)}\left(p^{\omega}_{md}+\frac{\partial p^{\omega}_{md}}{\partial x^{\omega}_{mfpd}}x^{\omega}_{mf(\omega)pd}\right)-\gamma^{\omega}_{mf(\omega)p}-\sum_{n}M3_{dn}\alpha p^{\omega}_{m,p,n}\leq 0 \tag{26}$$

Combining equations (24) and (26) and using the fact that $\forall n\in N,\ M2_{in}=M3_{dn}$ [8], we obtain:

$$\pi(\omega)\delta^{t(\omega)}\left(p^{\omega}_{md}+\frac{\partial p^{\omega}_{md}}{\partial x^{\omega}_{mfpd}}x^{\omega}_{mf(\omega)pd}-\eta_{pi}\right)+\eta p^{\omega}_{pi}\leq 0 \tag{27}$$

We know by equation (48e) and the fact that $up_{pi}>0$, that:

$$\eta_{pi}=\sum_{\omega}\eta p^{\omega}_{pi} \tag{28}$$

Therefore, since equation (27) is satisfied for all $\omega\in\Omega$, summing it over ω and using relation (28) gives:

$$\eta_{pi}\left(\sum_{\omega}\pi(\omega)\delta^{t(\omega)}-1\right)+\sum_{\omega}\pi(\omega)\delta^{t(\omega)}\left(p^{\omega}_{md}+\frac{\partial p^{\omega}_{md}}{\partial x^{\omega}_{mfpd}}x^{\omega}_{mf(\omega)pd}\right)\leq 0 \tag{29}$$

Since $\left(\sum_{\omega}\pi(\omega)\delta^{t(\omega)}-1\right)<0$, we have:

$$\eta_{pi}\geq\frac{-\sum_{\omega}\pi(\omega)\delta^{t(\omega)}\left(p^{\omega}_{md}+\frac{\partial p^{\omega}_{md}}{\partial x^{\omega}_{mfpd}}x^{\omega}_{mf(\omega)pd}\right)}{\left(\sum_{\omega}\pi(\omega)\delta^{t(\omega)}-1\right)}\geq 0 \tag{30}$$

□

The next theorem allows us to compare LTC and spot prices. Before, let us define the LTC constraints cost supported by an independent trader i. From the point of view of an independent trader, long-term contracts constrains him to purchase gas from the producers (he contracts with) each year, with a minimum proportional amount each season. In S-GaMMES, this is taken care of by constraints (20d) and (20f). Using the KKT conditions and the Lagrangian formulation, it is possible to define a cost inherent to the respect of these LTC constraints. Obviously, this cost that depends on the scenario node ω, the season m and the producer p involved in the contract, is function of the dual variables associated with constraints (20d) and (20f): ηi^{ω}_{pi} and υ^{ω}_{mpi}.

Definition 1. *The LTC cost between trader i and producer p is defined at each scenario node ω and each season m by:*

$$LTCcost^{\omega}_{mpi}=\eta i^{\omega}_{pi}-(1-min_{pi})\upsilon^{\omega}_{mpi}$$

[8] We recall that trader i is located at market d.

In the LTC cost definition, the term ηi^{ω}_{pi} takes care of the annual LTC constraint (i.e., the trader must purchase the same volume from the producer, at each scenario node) and the term $-(1 - min_{pi})\upsilon^{\omega}_{mpi}$ captures the seasonal LTC constraint (i.e., the trader must buy at least $100 \times min_{pi}$ percent of the annual LTC volume, at each season). Since the variable ηi^{ω}_{pi} is free (it is associated with an equality constraint), the LTC cost can be positive or negative.

In the following theorem and proof, we consider a particular pair of producer p and independent trader i who contract on the long-term. We will denote by d the consumption market where i is located.

Theorem 3. *If producer p and trader i contract on the long-term and $LTCcost^{\omega}_{mpi}$ is nonnegative then the spot price at market d is greater than the LTC price as long as trader i sells gas to market d:*

$$\forall \omega,\ m,\ y^{\omega}_{mid} > 0 \Longrightarrow p^{\omega}_{md} \geq \eta_{pi}$$

Proof. Producer p and trader i are assumed to contract on the long-term. Hence, $ui_{pi} > 0$. We already know, using equation (51c), that

$$\forall \omega, ui_{pi} = \sum_{m} zi^{\omega}_{mpi}$$

Thus, $\forall \omega$, $\exists m(\omega) \in M$ such as $zi^{\omega}_{mpi} > 0$. Because of equation (51e), the previous inequation holds for all the seasons:

$$\forall \omega, \forall m,\ zi^{\omega}_{mpi} > 0$$

Using equation (50a), it is possible to write: $\forall \omega, m$

$$-\pi(\omega)\delta^{t(\omega)}\eta_{pi} - \eta i^{\omega}_{pi} + \psi^{\omega}_{mi} + \sum_{n} M2_{in}\alpha i^{\omega}_{min} + (1 - min_{pi})\upsilon^{\omega}_{mpi} = 0 \tag{31}$$

If we assume that trader i sells gas to market d, then using equation (50b) and by denoting (for the sake of simplicity)

$$p^{\omega}_{md} + \frac{\partial p^{\omega}_{md}}{\partial y^{\omega}_{mid}} y^{\omega}_{mid}$$

the term

$$p^{\omega}_{md}(y^{\omega}_{mid} + \overline{y^{\omega}_{mid}}) + \frac{\partial p^{\omega}_{md}}{\partial y^{\omega}_{mid}}(y^{\omega}_{mid} + \overline{y^{\omega}_{mid}})y^{\omega}_{mid}$$

we find that:

$$\pi(\omega)\delta^{t(\omega)}\left(p^{\omega}_{md} + \frac{\partial p^{\omega}_{md}}{\partial y^{\omega}_{mid}} y^{\omega}_{mid}\right) - \psi^{\omega}_{mi} - \sum_{n} M3_{dn}\alpha i^{\omega}_{m,i,n} = 0 \tag{32}$$

Since trader i is located at market d, we can write:

$$\forall n \in N,\ M3_{dn} = M2_{in}$$

Combining equations (31) and (32), we find that:

$$\pi(\omega)\delta^{t(\omega)}\left(p^{\omega}_{md} + \frac{\partial p^{\omega}_{md}}{\partial y^{\omega}_{mid}} y^{\omega}_{mid} - \eta_{pi}\right) - \eta i^{\omega}_{pi} + (1 - min_{pi})\upsilon^{\omega}_{mpi} = 0 \tag{33}$$

or

$$\pi(\omega)\delta^{t(\omega)}\left(p^{\omega}_{md} + \frac{\partial p^{\omega}_{md}}{\partial y^{\omega}_{mid}} y^{\omega}_{mid} - \eta_{pi}\right) = \eta i^{\omega}_{pi} - (1 - min_{pi})\upsilon^{\omega}_{mpi} = LTCcost^{\omega}_{mpi} \tag{34}$$

In particular, if the LTC cost is nonnegative, we find that:

$$\pi(\omega)\delta^{t(\omega)}\left(p^{\omega}_{md}+\frac{\partial p^{\omega}_{md}}{\partial y^{\omega}_{mid}}y^{\omega}_{mid}-\eta_{pi}\right)\geq 0 \tag{35}$$

or

$$\pi(\omega)\delta^{t(\omega)}\left(p^{\omega}_{md}-\eta_{pi}\right)\geq -\pi(\omega)\delta^{t(\omega)}\left(\frac{\partial p^{\omega}_{md}}{\partial y^{\omega}_{mid}}y^{\omega}_{mid}\right) \tag{36}$$

Since the inverse demand function is decreasing, we can deduce that:

$$\frac{\partial p^{\omega}_{md}}{\partial y^{\omega}_{mid}}\leq 0 \tag{37}$$

Hence,

$$\pi(\omega)\delta^{t(\omega)}\left(p^{\omega}_{md}-\eta_{pi}\right)\geq -\pi(\omega)\delta^{t(\omega)}\left(\frac{\partial p^{\omega}_{md}}{\partial y^{\omega}_{mid}}y^{\omega}_{mid}\right)\geq 0 \tag{38}$$

and

$$\eta_{pi}\leq p^{\omega}_{md} \tag{39}$$

$\square$

From the point of view of an independent trader i, it may be interesting to study the variation of the LTC price among the different producers. Intuitively, since LTC prices are modeled as supply marginal costs for the trader i and since we assume that no market power is exerted by the producers on the LTC trade, we can deduce that all the producers will contract at the same price with i. The LTC price will therefore be correlated to the spot price because the latter is related to the profit earned by i, whereas the former is related to his supply cost. The following theorem details the relation between the different LTC prices.

Theorem 4. *If trader i contracts with producers p and p' on the long-term, then the LTC prices are equal:*

$$\eta_{pi}=\eta_{p'i}$$

Proof. We assume that trader i has LTCs with producers p and p', which means that $ui_{pi}>0$ and $ui_{p'i}>0$. To simplify the proof, we will assume that constraint (20f) is not binding. Therefore, the corresponding dual variables are such as:

$$\forall\omega,m,\ v^{\omega}_{mpi}=0$$

and

$$\forall\omega,m,\ v^{\omega}_{mp'i}=0$$

Let us demonstrate that $\eta_{pi}=\eta_{p'i}$.
Since $ui_{pi}>0$ and $ui_{p'i}>0$, we can use equation (50e) to deduce that

$$\sum_{\omega}\eta i^{\omega}_{pi}+\eta_{pi}=0 \tag{40}$$

and

$$\sum_{\omega}\eta i^{\omega}_{p'i}+\eta_{p'i}=0 \tag{41}$$

Since i contracted on the long-term with p and p', we can deduce, like in the previous proofs that:

$$\forall \omega,\ m,\ zi^{\omega}_{mpi} > 0 \text{ and } zi^{\omega}_{mp'i} > 0$$

Hence, using equation (50a), it is possible to write:

$$\forall \omega,\ m,\ -\pi(\omega)\delta^{t(\omega)}\eta_{pi} - \eta i^{\omega}_{pi} + \psi^{\omega}_{mi} + \sum_{n} M2_{in}\alpha i^{\omega}_{min} = 0 \tag{42}$$

and

$$\forall \omega,\ m,\ -\pi(\omega)\delta^{t(\omega)}\eta_{p'i} - \eta i^{\omega}_{p'i} + \psi^{\omega}_{mi} + \sum_{n} M2_{in}\alpha i^{\omega}_{min} = 0 \tag{43}$$

Summing equations (42) and (43) over ω and m and using relations (40) and (41), we can deduce that:

$$\left((\sum_{\omega} \pi(\omega)\delta^{t(\omega)}) - 1 \right) \eta_{pi} = \left((\sum_{\omega} \pi(\omega)\delta^{t(\omega)}) - 1 \right) \eta_{p'i} \tag{44}$$

or

$$\eta_{pi} = \eta_{p'i}$$

□

The following theorem concerns the stored volumes decided by the independent traders and the related reservation capacity.

Theorem 5. *The stored and reserved capacities for storage are such as:*

$$\forall \omega,\ \forall i,\ s,\ r^{\omega}_{is} > 0 \Rightarrow r^{\omega}_{is} = in^{\omega}_{is}$$

The previous theorem allows us to assert that at each scenario node, each storage site, the capacity reserved by an independent trader is always equal to the volume he actually decides to store. This result is very intuitive because the independent traders do not take care of storage investments. Hence, they are not affected, in their storage decision variables by the randomness of the demand. Theorem 5's proof is straightforward:

Proof. Let us assume that a trader i decides to make a storage reservation at storage site s: $r^{\omega}_{is} > 0$.
If he does not use completely the reserved capacity $r^{\omega}_{is} > in^{\omega}_{is}$, then using equation (51b), we deduce that:

$$\mu^{\omega}_{is} = 0 \tag{45}$$

If we consider relation (50c), we find that:

$$-\pi(\omega)\delta^{t(\omega)}Rc_s = \beta s^{\omega}_{s} \tag{46}$$

Since $\beta s^{\omega}_{s} \geq 0$ (using equation (54b)), we would have:

$$-\pi(\omega)\delta^{t(\omega)}Rc_s \geq 0 \tag{47}$$

which is absurd because $Rc_s > 0$.

□

3 Conclusion

This paper presents a Stochastic Generalized Nash-Cournot model in order to describe the natural gas markets' evolution, taking into account the fluctuations of the oil price. The demand representation takes into consideration the possible energy substitution that can be made between oil, coal, and natural gas. The exhaustivity of the resource is taken care of by the use of specific production cost functions (Golombek production cost functions).

The long-term contracts' prices and volumes are endogenously taken into account with the use of dual variables. This aspect makes our formulation a Generalized Nash-Cournot model, or similarly a QVI formulation. In order to solve it, we derived the VI formulation that usually presents a unique solution.

The demand is made random by considering the oil price's fluctuation with time. The model uses a scenario tree representation to capture the oil price fluctuation. The oil price's dynamic evolution is modeled as a Markov chain. The transition probabilities have been calibrated using an econometric study of the Brent price's historical evolution. The scenario tree representation allows us to not take care of non-anticipativity conditions. The consequence is that the model's formulation is very similar to the deterministic version GaMMES, with a bigger number of variables.

We have presented, proved and discussed a set of results and theorems related to our formulation. Most of these concern a comparison between the long-term contracts and spot markets gas prices. They allow one to understand the economic correlation between LTC and spot prices. Besides, when considering an independent trader, a comparison between all the LTC prices among all his possible supply sources is provided in order to understand the competition between the producers, in the upstream market.

S-GaMMES has been applied in order to study the evolution of the northwestern European gas trade. The results are gathered in another paper.

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4 Appendix 1

This appendix demonstrates the concavity of all the players' objective functions.
We will demonstrate that the production cost function is convex with respect to the quantity produced. The storage/withdrawal/investments costs are convex functions because they are linear.

Let's consider a producer p. First we demonstrate the convexity of the Golombek production cost function. We consider a production field f. To simplify the notation, let us denote by q the produced volume (a variable) and by Rf_f the reserve (a constant). We recall that the cost function Pc_f is as follows:

$$\begin{aligned} \tfrac{d\ Pc_f}{d\ q}: \quad & [0, Rf_f) \longrightarrow R^+ \\ & q \longrightarrow a_f + b_f q + \ c_f \ \ln\left(\tfrac{Rf_f - q}{Rf_f}\right) \end{aligned}$$

where $c_f \leq 0$ and $b_f \geq 0$.

Theorem 6. *The Golombek production cost function Pc_f is convex.*

Proof. Pc_f is a $C^2([0, Rf_f))$ function (twice continuously differentiable) and we have :

$$\forall q \in [0, Rf_f) \ \frac{d^2 Pc_f}{d^2 q} = b_f - \frac{c_f}{Rf_f - q} \geq 0$$

Thus, Pc_f is convex because $c_f \leq 0$ and $b_f \geq 0$.

□

Producer p's objective function is:
$\sum_{\omega,m,f,i} \pi(\omega)\delta^{t(\omega)}\eta_{pi}(zp^{\omega}_{mfpi})$
$+\sum_{\omega,m,f,d} \pi(\omega)\delta^{t(\omega)} \left(p^{\omega}_{md}(x^{\omega}_{mfpd} + \overline{x^{\omega}_{mfpd}})\right) x^{\omega}_{mfpd}$
$-\sum_{\omega,f} \delta^{t(\omega)} \left(Pc_f\left(\sum_{\omega' \leq \omega} \sum_m q^{\omega'}_{mfp}, Rf_f\right) - Pc_f\left(\sum_{\omega' < \omega} \sum_m q^{\omega'}_{mfp}, Rf_f\right)\right)$
$-\sum_{\omega,f} \delta^{t(\omega)} Ip_f(ip^{\omega}_{fp})$
$-\sum_{\omega,m,p,a} \delta^{t(\omega)}((Tc_a + \tau^{\omega}_{m,a}) fp^{\omega}_{m,p,a})$

Theorem 7. *Producer p's objective function is concave with respect to his decision variables.*

Proof. As mentioned before, the inverse demand function has been linearized. Let's write the natural gas price in market d, season m and node ω as follows:

$$p^{\omega}_{md} = a^{\omega}_{md} - b^{\omega}_{md}(x^{\omega}_{mfpd} + \overline{x^{\omega}_{mfpd}})$$

where $b^{\omega}_{md} > 0$. The function $\sum_{\omega,m,f,d} \pi(\omega)\delta^{t(\omega)} \left(p^{\omega}_{md}(x^{\omega}_{mfpd} + \overline{x^{\omega}_{mfpd}})\right) x^{\omega}_{mfpd}$ is therefore a concave function of the variables x^{ω}_{mfpd}. Indeed, the Hessian matrix H^{ω}_{md} associated with the spot market profit is diagonal and such that the diagonal terms are $H^{\omega}_{md} = -2b^{\omega}_{md} < 0$. Hence, the Hessian matrix is negative definite.

Let us consider the global cost function GP:
$q^{\omega}_{mfp} \longrightarrow GP(q^{\omega}_{mfp}) = \sum_{\omega,f} \delta^{t(\omega)} \left(Pc_f\left(\sum_{\omega' \leq \omega} \sum_m q^{\omega'}_{mfp}, Rf_f\right) - Pc_f\left(\sum_{\omega' < \omega} \sum_m q^{\omega'}_{mfp}, Rf_f\right)\right)$.
And let's demonstrate that GP is convex. Let us consider two variable vectors $q1^{\omega}_{md}$ and $q2^{\omega}_{md}$

and $\lambda \in [0, 1]$.
We denote by Ω_l the subset of Ω that contains all the leaves of the tree.

$$
\begin{aligned}
&GP(\lambda q1_{md}^{\omega} + (1-\lambda) q2_{md}^{\omega}) \\
&= \\
&\sum_{\omega,f} \delta^{t(\omega)} \left(Pc_f \left(\sum_{\omega' \leq \omega} \sum_m (\lambda q1_{md}^{\omega'} + (1-\lambda) q2_{md}^{\omega'}), Rf_f \right) \right) \\
&- \sum_{\omega,f} \delta^{t(\omega)} \left(Pc_f \left(\sum_{\omega' < \omega} \sum_m (\lambda q1_{md}^{\omega'} + (1-\lambda) q2_{md}^{\omega'}), Rf_f \right) \right) \\
&= \\
&\sum_f \sum_{\omega \in \Omega} \delta^{t(\omega)} \left(Pc_f \left(\sum_{\omega' \leq \omega} \sum_m (\lambda q1_{md}^{\omega'} + (1-\lambda) q2_{md}^{\omega'}), Rf_f \right) \right) \\
&- \sum_f \sum_{\omega \notin \Omega_l} \delta^{t(\omega)+1} \left(Pc_f \left(\sum_{\omega' \leq \omega} \sum_m (\lambda q1_{md}^{\omega'} + (1-\lambda) q2_{md}^{\omega'}), Rf_f \right) \right) \\
&= \\
&\sum_f \sum_{\omega \notin \Omega_l} (\delta^{t(\omega)} - \delta^{t(\omega)+1}) \left(Pc_f \left(\sum_{\omega' \leq \omega} \sum_m (\lambda q1_{md}^{\omega'} + (1-\lambda) q2_{md}^{\omega'}), Rf_f \right) \right) \\
&+ \sum_f \sum_{\omega \in \Omega_l} \delta^{Num} \left(Pc_f \left(\sum_{\omega' \leq \omega} \sum_m (\lambda q1_{md}^{\omega'} + (1-\lambda) q2_{md}^{\omega'}), Rf_f \right) \right) \\
&= \\
&\sum_f \sum_{\omega \notin \Omega_l} \delta^{t(\omega)} (1-\delta) \left(Pc_f \left(\sum_{\omega' \leq \omega} \sum_m (\lambda q1_{md}^{\omega'} + (1-\lambda) q2_{md}^{\omega'}), Rf_f \right) \right) \\
&+ \sum_f \sum_{\omega \in \Omega_l} \delta^{Num} \left(Pc_f \left(\sum_{\omega' \leq \omega} \sum_m (\lambda q1_{md}^{\omega'} + (1-\lambda) q2_{md}^{\omega'}), Rf_f \right) \right)
\end{aligned}
$$

Since $0 \leq \delta \leq 1$ and Pc_f is convex, we can write:

$$
\begin{aligned}
&\sum_f \sum_{\omega \notin \Omega_l} \delta^{t(\omega)} (1-\delta) \left(Pc_f \left(\sum_{\omega' \leq \omega} \sum_m (\lambda q1_{md}^{\omega'} + (1-\lambda) q2_{md}^{\omega'}), Rf_f \right) \right) \\
&+ \sum_f \sum_{\omega \in \Omega_l} \delta^{Num} \left(Pc_f \left(\sum_{\omega' \leq \omega} \sum_m (\lambda q1_{md}^{\omega'} + (1-\lambda) q2_{md}^{\omega'}), Rf_f \right) \right) \\
&\leq \\
&\lambda \sum_f \sum_{\omega \notin \Omega_l} \delta^{t(\omega)} (1-\delta) \left(Pc_f \left(\sum_{\omega' \leq t} \sum_m q1_{md}^{\omega'}, Rf_f \right) \right) \\
&+ (1-\lambda) \sum_f \sum_{\omega \notin \Omega_l} \delta^{t(\omega)} (1-\delta) \left(Pc_f \left(\sum_{\omega' \leq t} \sum_m q2_{md}^{\omega'}, Rf_f \right) \right) \\
&+ \lambda \sum_f \sum_{\omega \in \Omega_l} \delta^{Num} \left(Pc_f \left(\sum_{\omega' \leq \omega} \sum_m q1_{md}^{\omega'}, Rf_f \right) \right) \\
&+ (1-\lambda) \sum_f \delta^{Num} \left(Pc_f \left(\sum_{\omega' \leq \omega} \sum_m q2_{md}^{\omega'}, Rf_f \right) \right) \\
&= \\
&\lambda GP(q1_{md}^{\omega}) + (1-\lambda) GP(q2_{md}^{\omega})
\end{aligned}
$$

Hence, the cost function is convex. The rest of the profit is made of linear functions of the decision variables. The concavity of the producers' objective function is thus demonstrated. □

Theorem 8. *The independent traders' objective function is concave with respect to his decision variables.*

Theorem 9. *The pipeline and storage operators objective functions are convex.*

Theorem 10. *All the players' constraint sets are convex.*

Proof. The proof of the independent traders' concavity of their objective function is similar to the previous proof. Like for the producers, the spot market benefit is in particular concave.
The pipeline and storage operators objective functions are convex because they are linear.
The feasibility sets are all convex due to linearity of the constraint functions. □

5 Appendix 2

This appendix presents the KKT conditions derived from S-GaMMES. Once the KKT conditions written, we get the Mixed Complementarity Problem (MCP) given below.

The producers' KKT conditions

$$\forall \omega,\ m,\ f,\ p,\ i, \quad 0 \le zp^{\omega}_{mfpi} \quad \perp \quad \pi(\omega)\delta^{t(\omega)}\eta_{pi} - \gamma^{\omega}_{mfp} - \epsilon 2^{\omega}_{mfpi} - \eta p^{\omega}_{pi} - \sum_n M2_{in}\alpha p^{\omega}_{m,p,n} \le 0 \tag{48a}$$

$$\forall \omega,\ m,\ f,\ p,\ d, \quad 0 \le x^{\omega}_{mfpd} \quad \perp \quad \pi(\omega)\delta^{t(\omega)} p^{\omega}_{md}(x^{\omega}_{mfpd} + \overline{x^{\omega}_{mfpd}}) + \pi(\omega)\delta^{t(\omega)}\frac{\partial p^{\omega}_{md}}{\partial x^{\omega}_{mfpd}}(x^{\omega}_{mfpd} + \overline{x^{\omega}_{mfpd}})x^{\omega}_{mfpd} - \gamma^{\omega}_{mfp} - \epsilon 1^{\omega}_{mfpd} - \sum_n M3_{dn}\alpha p^{\omega}_{m,p,n} \le 0 \tag{48b}$$

$$\forall \omega,\ m,\ f,\ p, \quad 0 \le q^{\omega}_{mfp} \quad \perp \quad -\sum_{\omega' \ge \omega} \pi(\omega')\delta^{t(\omega')}\frac{\partial Pc_f}{\partial q}\Big(\sum_{\omega'' \le \omega'}\sum_m q^{\omega''}_{mfp}, Rf_f\Big) + \sum_{\omega' > \omega} \pi(\omega')\delta^{t(\omega')}\frac{\partial Pc_f}{\partial q}\Big(\sum_{\omega'' < \omega'}\sum_m q^{\omega''}_{mfp}, Rf_f\Big) - \sum_{\omega' \ge \omega}\phi^{\omega'}_f - \chi^{\omega}_{mf} + \gamma^{\omega}_{mfp} - (-1)^m(\vartheta 1^{\omega}_{fp} - \vartheta 2^{\omega}_{fp}) - \epsilon 3^{\omega}_{mfp} + \sum_n M1_{fn}\alpha p^{\omega}_{m,p,n} \le 0 \tag{48c}$$

$$\forall \omega,\ f,\ p, \quad 0 \le ip^{\omega}_{fp} \quad \perp \quad -\pi(\omega)\delta^{t(\omega)} Ip_f - \epsilon 4^{\omega}_{fp} + \sum_m \sum_{\omega' > \omega} \chi^{\omega'}_{mf}(1 - dep_f)^{t(\omega') - t(\omega)} - \iota p^{\omega}_f + Lf_f \sum_{\omega' > \omega} \iota p^{\omega'}_f (1 - dep_f)^{t(\omega') - t(\omega)} \le 0 \tag{48d}$$

$$\forall\ p,\ i, \quad 0 \le up_{pi} \quad \perp \quad \sum_{\omega} \eta p^{\omega}_{pi} - \eta_{pi} \le 0 \tag{48e}$$

$$\forall \omega,\ f, \quad 0 \le \phi^{\omega}_f \quad \perp \quad \sum_p \sum_{\omega' \le \omega} \sum_m q^{\omega'}_{mfp} - Rf_f \le 0 \tag{48f}$$

$$\forall \omega,\ m,\ f, \quad 0 \le \chi^{\omega}_{mf} \quad \perp \quad \sum_p q^{\omega}_{mfp} - Tk_f(1 - dep_f)^{t(\omega)} - \sum_p \sum_{\omega' < \omega} ip^{\omega'}_{fp}(1 - dep_f)^{t(\omega) - t(\omega')} \le 0 \tag{48g}$$

$$\forall \omega,\ m,\ f,\ p, \qquad 0 \leq \gamma^{\omega}_{mfp} \quad \perp \quad -q^{\omega}_{mfp} + \sum_i zp^{\omega}_{mfpi} + \sum_d x^{\omega}_{mfpd} \leq 0 \tag{49a}$$

$$\forall \omega,\ f,\ p, \qquad 0 \leq \vartheta 1^{\omega}_{fp} \quad \perp \quad \sum_m (-1)^m q^{\omega}_{mfp} - fl_f \leq 0 \tag{49b}$$

$$\forall \omega,\ f,\ p, \qquad 0 \leq \vartheta 2^{\omega}_{fp} \quad \perp \quad -\sum_m (-1)^m q^{\omega}_{mfp} - fl_f \leq 0 \tag{49c}$$

$$\begin{aligned}\forall t,\ f, \qquad 0 \leq \iota p^{\omega}_{f} \quad \perp \quad & \sum_p ip^{\omega}_{fp} - Lf_f Kf_f (1 - dep_f)^{t(\omega)} \\ & - Lf_f \sum_p \sum_{\omega' < \omega} ip^{t(\omega')}_{fp} (1 - dep_f)^{t(\omega) - t(\omega')} \leq 0\end{aligned} \tag{49d}$$

$$\forall \omega,\ f, m,\ p,\ d, \qquad 0 \leq \epsilon 1^{\omega}_{mfpd} \quad \perp \quad x^{\omega}_{mfpd} - O_{fp} H \leq 0 \tag{49e}$$

$$\forall \omega,\ m, f,\ p,\ i, \qquad 0 \leq \epsilon 2^{\omega}_{mfpi} \quad \perp \quad zp^{\omega}_{mfpi} - O_{fp} H \leq 0 \tag{49f}$$

$$\forall \omega,\ m, f,\ p, \qquad 0 \leq \epsilon 3^{\omega}_{mfp} \quad \perp \quad q^{\omega}_{mfp} - O_{fp} H \leq 0 \tag{49g}$$

$$\forall \omega,\ f,\ p, \qquad 0 \leq \epsilon 4^{\omega}_{fp} \quad \perp \quad ip^{\omega}_{fp} - O_{fp} H \leq 0 \tag{49h}$$

$$\forall \omega,\ p,\ i, \qquad \text{free} \quad \eta p^{\omega}_{pi} \qquad up_{pi} - \sum_{f,m} zp^{\omega}_{mfpi} = 0 \tag{49i}$$

$$\forall\ p,\ i, \qquad \text{free} \quad \eta_{pi} \qquad ui_{pi} - up_{pi} = 0 \tag{49j}$$

The independent traders' KKT conditions

$$\forall \omega,\ m,\ p,\ i, \quad 0 \leq zi^{\omega}_{mpi} \quad \perp \quad \begin{aligned} & -\pi(\omega)\delta^{t(\omega)}\eta_{pi} - \eta i^{\omega}_{pi} \\ & + \psi^{\omega}_{mi} \\ & + \sum_n M2_{in}\alpha i^{\omega}_{min} \\ & + (1 - min_{pi})\upsilon^{\omega}_{mpi} \end{aligned} \quad \leq 0 \tag{50a}$$

$$\forall \omega,\ m,\ i,\ d, \quad 0 \leq y^{\omega}_{mid} \quad \perp \quad \begin{aligned} & \pi(\omega)\delta^{t(\omega)}p^{\omega}_{md}(y^{\omega}_{mid} + \overline{y^{\omega}_{mid}}) \\ & + \pi(\omega)\delta^{t(\omega)}\frac{\partial p^{\omega}_{md}}{\partial y^{\omega}_{mid}}(y^{\omega}_{mid} + \overline{y^{\omega}_{mid}})y^{\omega}_{mid} \\ & - \psi^{\omega}_{mi} - \sum_n M3_{dn}\alpha i^{\omega}_{m,i,n} \end{aligned} \quad \leq 0 \tag{50b}$$

$$\forall \omega,\ i,\ s, \quad 0 \leq r^{\omega}_{is} \quad \perp \quad -\pi(\omega)\delta^{t(\omega)}Rc_s + \mu^{\omega}_{is} - \beta s^{\omega}_s \quad \leq 0 \tag{50c}$$

$$\forall \omega,\ i,\ s, \quad 0 \leq in^{\omega}_{is} \quad \perp \quad \begin{aligned} & -\pi(\omega)\delta^{t(\omega)}(Ic_s + Wc_s) \\ & - \mu^{\omega}_{is} - \sum_m (-1)^m \psi^{\omega}_{mi} \\ & - \sum_n M4_{sn}\alpha i^{\omega}_{m,i,n}(-1)^m \end{aligned} \quad \leq 0 \tag{50d}$$

$$\forall p,\ i, \quad 0 \leq ui_{pi} \quad \perp \quad \sum_{\omega} \eta i^{\omega}_{pi} + \eta_{pi} \quad \leq 0 \tag{50e}$$

$$\forall \omega,\ m,\ i, \quad \text{free} \quad \psi^{\omega}_{mi} \quad \sum_p zi^{\omega}_{mpi} - \sum_d y^{\omega}_{mid} + (-1)^m \sum_s in^{\omega}_{is} \quad = 0 \tag{51a}$$

$$\forall \omega,\ i,\ s, \quad 0 \leq \mu^{\omega}_{is} \quad \perp \quad in^{\omega}_{is} - r^{\omega}_{is} \quad \leq 0 \tag{51b}$$

$$\forall \omega,\ p,\ i, \quad \text{free} \quad \eta i^{\omega}_{pi} \quad ui_{pi} - \sum_m zi^{\omega}_{mpi} \quad = 0 \tag{51c}$$

$$\forall p,\ i, \quad \text{free} \quad \eta_{pi} \quad ui_{pi} - up_{pi} \quad = 0 \tag{51d}$$

$$\forall \omega,\ m,\ p,\ i, \quad 0 \leq \upsilon^{\omega}_{mpi} \quad \perp \quad -zi^{\omega}_{mpi} + min_{pi}\sum_m zi^{\omega}_{mpi} \quad \leq 0 \tag{51e}$$

The pipeline operator's KKT conditions

$$\forall \omega,\ m,\ p,\ a, \qquad 0 \leq fp^{\omega}_{m,p,a} \quad \perp \quad -\pi(\omega)\delta^{t(\omega)}(Tc_a + \tau^{\omega}_{m,a}) - \tau^{\omega}_{m,a} + \sum_n M6_{an}\alpha p^{\omega}_{p,m,n}(1 - loss_a) - \sum_n M5_{an}\alpha p^{\omega}_{p,m,n} \leq 0 \tag{52a}$$

$$\forall \omega,\ m,\ i,\ a, \qquad 0 \leq fi^{\omega}_{m,i,a} \quad \perp \quad -\pi(\omega)\delta^{t(\omega)}(Tc_a + \tau^{\omega}_{m,a}) - \tau^{\omega}_{m,a} + \sum_n M6_{an}\alpha i^{\omega}_{i,m,n}(1 - loss_a) - \sum_n M5_{an}\alpha i^{\omega}_{i,m,n} \leq 0 \tag{52b}$$

$$\forall \omega,\ a, \qquad 0 \leq ik^{\omega}_{a} \quad \perp \quad -\pi(\omega)\delta^{t(\omega)} Ik_a + \sum_{\omega' > \omega} \tau^{\omega'}_{m,a} - \iota a^{\omega}_a + La_a \sum_{\omega' > \omega} \iota a^{\omega'}_a \leq 0 \tag{52c}$$

$$\forall \omega,\ m,\ a, \qquad 0 \leq \tau^{\omega}_{m,a} \quad \perp \quad \sum_p fp^{\omega}_{m,p,a} + \sum_i fi^{\omega}_{m,i,a} - Tk_a - \sum_{\omega' < \omega} ik^{\omega'}_a \leq 0 \tag{52d}$$

$$\forall \omega,\ a, \qquad 0 \leq \iota a^{\omega}_{a} \quad \perp \quad ik^{\omega}_a - Tk_a - \sum_{\omega' < \omega} ik^{\omega}_a \leq 0 \tag{52e}$$

$$\forall \omega,\ m,\ p,\ n, \qquad \text{free} \quad \alpha p^{\omega}_{m,p,n} \qquad \sum_a M_6(a,n) fp^{\omega}_{m,p,a}(1 - loss_a) - \sum_a M5_{an} fp^{\omega}_{m,p,a} + \sum_f q^{\omega}_{mpf} M1_{fn} - \sum_d \sum_f x^{\omega}_{mfpd} M3_{dn} - \sum_i \sum_f zp^{\omega}_{mfpi} M2_{in} = 0 \tag{52f}$$

$$\forall\omega,\ m,\ i,\ n, \quad \text{free} \quad \alpha i^{\omega}_{m,i,n} \quad \sum_a M6_{an} fi^{\omega}_{m,i,a}(1-loss_a) - \sum_a M5_{an} fi^{\omega}_{m,i,a} - \sum_d y^{\omega}_{mpd} M3_{dn} + \sum_p zi^{\omega}_{mpi} M2_{in} - (-1)^m \sum_s \sum_i in^{\omega}_{is} M4_{sn} = 0 \tag{53a}$$

The storage operator's KKT conditions

$$\forall\omega,\ s, \quad 0 \le is^{\omega}_s \quad \perp \quad -\pi(\omega)\delta^{t(\omega)} Is_s + \sum_{\omega'>\omega} \beta s^{\omega'}_s - \iota s^{\omega}_s + Ls_s \sum_{\omega'>\omega} \iota s^{\omega'}_s \le 0 \tag{54a}$$

$$\forall\omega,\ s, \quad 0 \le \beta s^{\omega}_s \quad \perp \quad \sum_i r^{\omega}_{is} - Ks_s - \sum_{\omega'<\omega} is^{\omega'}_s \le 0 \tag{54b}$$

$$\forall\omega,\ s, \quad 0 \le \iota s^{\omega}_s \quad \perp \quad is^{\omega}_s - Ls_s Ks_s - Ls_s \sum_{\omega'<\omega} is^{\omega'}_s \le 0 \tag{54c}$$

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