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JEL Classification: C61, C73, D24,D43, L13, Q41

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Abstract

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*All proofs are available on request.

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1 Introduction

The economic literature provides an important panel of numerical models whose objective is to describe the natural gas trade structure. Most of these contributions, like (8), (22), (23), (25), (26) do not take into account fuel substitution. Besides, standard stochastic natural gas market models, like the "Stochastic World Gas Model" (University of Maryland) (9) or (12), (29) and (18) usually represent stochasticity of the demand. In most of these models, the demand function is represented as an affine function with a stochastic intercept whose probability law is discrete. This approach leads to the construction of a scenario tree that captures the stochastic dynamics of the market. Unfortunately most of these approaches give arbitrary probability laws to the demand levels and do not carry out a realistic calibration process. As an example, the stochastic parameter may follow a Gaussian distribution with an arbitrary mean value and variance (9).

Our model, named S-GaMMES, Stochastic Gas Market Modeling with Energy Substitution, is a Stochastic Generalized-Nash Cournot model that describes the natural gas market trade in Europe. Its key features are the following: the strategic interaction (market power exercise) between the players is captured, energy substitution between coal, oil, and natural gas is taken into account, long-term contracts (LTC) are endogenously described and the oil price's fluctuation is captured by modeling the oil price as a random variable. The major gas chain players are depicted including: producers, consumers, storage, and pipeline operators, as well as intermediate local traders [1].

Stochastic models are often huge in terms of number of variables and hold computational problems when solving them, which forces the modeler to use decomposition techniques, such as the Benders' decomposition (3), (13) or scenario reduction methods (7). Therefore, one must select carefully the type of randomness (production, demand, etc.) to consider (21).

Among all the types of random gas market's characteristics, we decided to model the uncertainty associated with the demand because on the one hand we believe that its impact on the markets' outcome (especially prices and consumption) is the most important and, on the other hand, the demand function specification is the most serious drawback of current gas markets models (27), because it presents an arbitrary aspect in the calibration.

[1]Throughout this paper, what we call independent traders are actually midstreamers such as the French GDF SUEZ or the Italian ENI

A casual look at the oil and gas prices in the spot markets suggests that they are strongly correlated (20). This is mainly due to two reasons: the first is the Long-Term Contract (LTC) gas prices' oil price indexation and the second is related to energy substitution.
LTC prices between producers and traders have always been indexed by the oil products' price, to allow natural gas to be a competitive fuel [2]. Since LTC prices constitute a supply marginal cost for the traders, they are correlated to the gas spot prices and, therefore, spot gas and oil prices become correlated too.
Energy substitution also plays an important role in linking the fuels' prices. Indeed, if the consumers are allowed to choose their energy consumption's source, they will go for the cheapest fuel to satisfy their demand (notwithstanding capacity consumption and investment inertia). Therefore, such a consumption feature will ensure all the fuels remain competitive in the market and will link their prices.
Taking into account long-term contract oil price's indexation in gas markets modeling requires exogenous data, such as the indexation formula between each pair of producer/trader. Because of a lack of such data, we decided to focus mainly on energy substitution to capture the gas and oil prices correlation.

The interaction between all the players is a Generalized Nash-Cournot competition and we explicitly take into consideration, in an endogenous way, the long-term contractual aspects (prices and volumes) of the markets that link the producers and the traders. The representation of the demand includes the possible substitution, within the overall energy consumption, between different types of fuels. Hence, in our work, we mitigate the market power exerted by the strategic players: they cannot force the natural gas price up freely because some consumers would switch to other fuels to satisfy their demand.
The specification of the demand function is the one derived from the system dynamics approach presented in (2). Besides, in order to capture the oil price's fluctuation and the oil/gas price correlation, we decided to model the oil price as a random variable. This property makes the demand function stochastic.

[2]Recently, most of the European midstreamers have pushed to index part of their contracts on the spot prices when renegotiating.

The remaining parts of this paper are as follows: Section 2 gives a general description of the chosen economic structure representation and explains the stochastic oil price effects through a scenario tree. In Section 3, we propose an application of our model to the northwestern European natural gas trade and we calculate the value of the stochastic solution. The last section concludes the paper.

2 The model

2.1 Market structure

We consider, in an endogenous way, long-term contracts between the independent traders and the producers[3]. We divide the markets in two stages: the upstream part that represents production and the downstream one, constituted by the different spot markets (end-use consumption markets). Both stages are linked by a set of independent traders. The traders buy gas from the producers on a long-term contract basis and bring it to the spot markets where market power is exerted. Both producers and traders exert market power in the spot markets and compete via a Nash-Cournot competition. Long-term contracts, production, transportation, and storage investments are endogenous (in terms of prices and volumes) to the model and this property makes our formulation a Generalized Nash-Cournot game.

Considering energy substitutions in the natural gas demand mitigates the market power that can be exerted by the strategic players (producers and traders) in the end-use markets. Indeed, this is due to the fact that the consumers have the ability to reduce the natural gas share in their energy mix if the gas market price is much higher than the substitution fuels' (such as oil and coal) price. Therefore, the producers may not have any considerable incentive to reduce their natural gas production in order to force the price up. This model property allows us to take into account the oil/natural gas prices correlation: the Nash-Cournot interaction will link the natural gas price to the coal and oil prices because of the demand function dependence on these parameters.

In order to capture the demand fluctuations, we introduced stochasticity the fluctuations of the oil price. For that purpose, an econometric study of the oil price is carried out in order to deduce

[3]See (1) for more details about the economic structure.

and calibrate the probability law of the oil price's dynamic evolution. Studying the impact of the oil price's fluctuation over the gas markets is possible because we have modeled fuel substitution when representing the gas demand.

The model also takes advantage of a scenario tree representation where each node represents the intersection of randomness and time. The oil price, at each time-step, is hence a random variable that influences the demand function at each scenario node.

The transport and storage infrastructure is modeled using competitive pipeline and storage operators whose objective is to minimize the capital and operational costs. Regarding the transport, the cost includes transportation, congestion, and investment fees. Regarding the storage, the cost includes capacity reservation, storage, withdrawal, and investment fees.

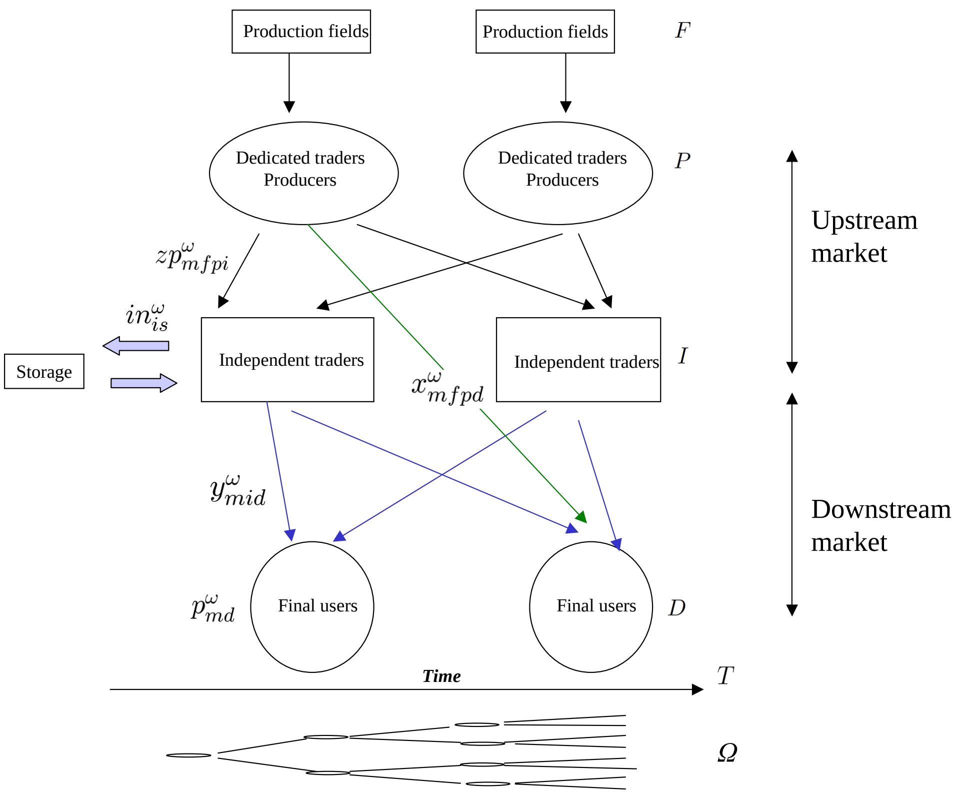

Figure 1:
The market representation

2.2 Agents' behavior

- The producers are risk neutral and strive to maximize their profit's expectation. They can invest to increase their production capacity and can sell their production to the spot markets or to the midstreamers on a long-term contract basis. They can exert market power only in the spot markets.

- The independent traders (or midstreamers) are intermediaries between the upstream stage and consumers. They buy gas from the producers on the long-term contracts and sell it in the spot markets, where they can also exert market power. Therefore, they are in competition in the hubs with the producers from which they buy gas on the contracts. They maximize their expected revenues and are risk neutral.

- Consumers are represented by the inverse demand function in each consumption node (or spot market). The calculation of the inverse demand function is based on a System Dynamics approach developped in (2) where inter-fuel substitution is explicitly modeled. The competition between the producers and independent traders is of the Cournot type. Hence, the model will determine at the same time both the gas consumption and prices in the spot markets.

- Long-term contracts between a pair of producer/independent trader are such that the Take Or Pay volume [4] and the contract price are constant over time [5]. They are modeled as follows: the producer decides the supply he wants to sell on the contract and the trader the volume he demands. An LTC supply = demand constraint is added to clear the LTC market and its dual variable represents the contract price. Thus, both the LTC Take Or Pay volume and price are endogenously modeled.

- The transport (TSO) and pipeline operators (SSO) are regulated: they operate the infrastructure so that the total investment and operations costs are minimal and the demand is met.

We believe that combining the endogenous representation of long-term contracts, energy substitution and stochasticity of the demand in our model is a novelty in partial equilibrium gas

[4]The Take Or Pay volume is the minimum volume of the contract that the trader has to pay for, even if such a volume is not entirely needed.

[5]In other words, we only model endogenously forward contracts.

markets modeling with imperfect competition.

We now focus on the demand's stochastic tree representation.

2.3 The scenario tree

We refer to (2) for a detailed description of our demand representation, where it is explained how the oil price influences the gas demand function. In this section, we detail how the oil price is modeled as a random variable, which is the main improvement of this work with respect to the deterministic version of the model.

The demand is made stochastic in order to capture the strong fluctuations of the oil price in Europe. The oil price's dynamic evolution will influence the inverse demand function parameters. Indeed, if the oil price is high in a certain year, consumers will invest more in natural gas burners (the substitute) and therefore, the future demand for natural gas will rise [6]. On the contrary, a low oil price will reduce the future demand for natural gas. The study of the coal price's evolution over time indicates that its fluctuation is negligible compared to the oil one (5). Therefore, the coal price is not taken as random.
Let us denote by p_b the chain of the Brent price, with a six-month time-step [7] and ζ_b the corresponding logarithmic percentage price change:

$$\zeta_b = \frac{\ln(p_{b+1}) - \ln(p_b)}{\ln(p_b)} \tag{1}$$

The data base we use for the Brent price is given in (5). More precisely, p_b is the mean value, over six months, of the Brent price and ζ_b the six-month logarithmic percentage change. We consider the evolution of the price p_b and ζ_b, $b \in \{1, 2, ...64\}$. $b = 1$ corresponds to the period July 1977 to December 1977 and $b = 64$ to the period January 2009 to June 2009. Figure 2 is a histogram of the variable ζ_b fitted with a Gaussian distribution.

The statistical study we carried out provided a normalized error of 0.2 (see later for the error's definition), for the Gaussian fit shown in Figure 2. The other numerical results (mean

[6] This argument holds for a constant evolution of the coal price.
[7] This time-step is the one that gives the best results in our study.

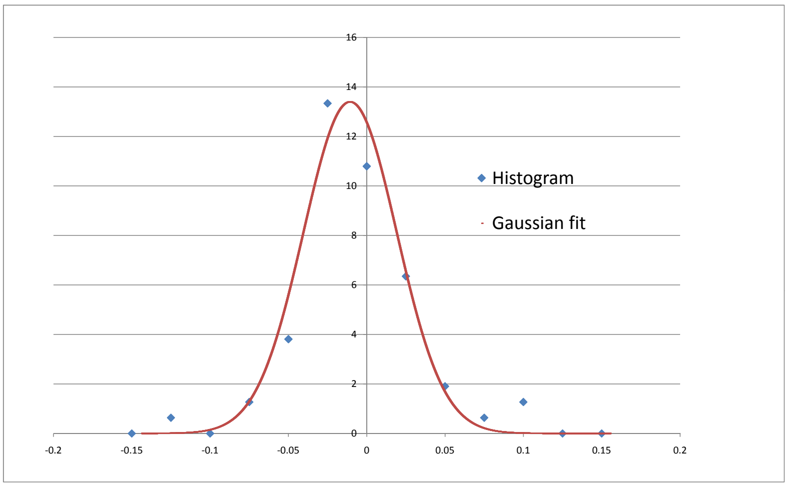

Figure 2:
The histogram of ζ_b and the Gaussian fit.

value, variance) will be provided later.

A visual inspection of the correlogram shows no sign of linear auto-correlation between the variables ζ_b, $b \in \{1, 2, ...64\}$. In addition, the variables' independence has been checked using the BDS test (6) (the BDS statistics with two dimensions 0.008 with probability 0.52). The ζ_b variables can therefore be considered as independent and identically distributed random variables. The Kolmogorov-Smirnov (24) test allows us to state that they have a normal distribution. Indeed, the test did not reject the 0-hypothesis of normality (Adj. value 1.04 with probability 0.22). The Gaussian fit is obtained by minimizing the normalized error between a Gaussian distribution and the histogram points of ζ_b. The normalized error is given by the following: if (x_i, y_i), $i \in \{1, 2...n\}$ are the histogram points and $\mathcal{N}_{x0,\sigma}$ a Gaussian distribution, the error $e_{x0,\sigma}$ is:

$$e_{x0,\sigma} = \frac{1}{n}\sum_{i=1}^{n} |\frac{y_i - \mathcal{N}_{x0,\sigma}(x_i)}{y_i}| \tag{2}$$

In the representation of the European natural gas trade, we may need to use a time-step longer than six months [8]. Hence, it is worthwhile to explain how we can deduce the new log-

[8] Like in the deterministic version, we typically use a five-year time-step in the stochastic version.

percentage change's probability density that can be used directly by the model. Let us assume that the model's study time-step is $\kappa\times$ 6 months where $\kappa \in \mathbb{N}$, and call λ the new log-percentage change:

$$\lambda_b = \frac{\ln(p_{b+\kappa}) - \ln(p_b)}{\ln(p_b)} \tag{3}$$

λ_b takes into account the $\kappa\times 6$ months offset. In our case, κ=10 (relation between five years and six moinths time-steps). The relationship between λ_b and ζ_b is given using the following lemmas and theorem:

Lemma 1. $\forall b \in \mathbb{N},\ p_{b+\kappa} = p_b^{\Pi_{i=0}^{\kappa-1}(1+\zeta_{b+i})}$

The previous equation can be rewritten as follows:

$$\ln(p_{b+\kappa}) = \Pi_{i=0}^{\kappa-1}(1+\zeta_{b+i})\ \ln(p_b) \tag{4}$$

Figure 2 indicates that the random variable ζ is such that $|\zeta| \leq 0.05$ with a more than 90% probability (this can be verified easily). Hence, we can write that, in first approximation, $\forall b \in \mathbb{N}, \zeta_b << 1$ and

$$\Pi_{i=0}^{\kappa-1}(1+\zeta_{b+i}) \simeq 1 + \sum_{i=0}^{\kappa-1} \zeta_{b+i} \tag{5}$$

The approximation error can be bounded via the following theorem:

Theorem 1. *If we denote by* $\epsilon = \Pi_{i=0}^{\kappa-1}(1+\zeta_{b+i}) - \left(1 + \sum_{i=0}^{\kappa-1}\zeta_{b+i}\right)$ *the error and* ζ_{max} *the maximum absolute value of* ζ_k*:* $\zeta_{max} = Max\{|\zeta_k|,\ k \in \{b, ..., b+\kappa-1\}\}$*, then:*

$$|\epsilon| \leq (1+\zeta_{max})^{\kappa} - 1 - \kappa\zeta_{max} \tag{6}$$

Theorem 1 allows us to state that approximation 5 is valid with an error of 10%, with a 90% probability. Indeed, as said before, Figure 2 shows that the random variable ζ_{max} is such that $\zeta_{max} \leq 0.05$ with a more than 90% probability. Therefore, $|\epsilon| \leq 0.1$ with more than 90% probability. Using equations (4) and (5), we can deduce that:

$$\lambda_b = \sum_{i=0}^{\kappa-1} \zeta_{b+i} \tag{7}$$

Since we assumed that ζ_b are independent and identically distributed random variables and since we know that they follow the Gaussian distribution $\mathcal{N}_{x_0,\sigma}$, then we can derive that λ_b are also independent and identically distributed and follow a Gaussian probability distribution $\mathcal{N}_{\kappa x_0,\sqrt{\kappa}\sigma}$.

In order to solve the model in a reasonable time, we decided to use only two scenarios for the oil price at each time-step raher than keeping the continuous Gaussian probability law. Therefore, we have to approximate the logarithmic yield λ's Gaussian probability density $\mathcal{N}_{\kappa x_0,\sqrt{\kappa}\sigma}$ by a two-value probability law. Let us call λ_1 and λ_2 the two possible values of the random variable λ that will be used by the model, θ and $1-\theta$ the associated probabilities. The goal now is to find λ_1, λ_2 and θ.
The mean value and the standard deviation of λ are respectively κx_0 and $\sqrt{\kappa}\sigma$. Therefore, we can write:

Lemma 2. *λ_1, λ_2 and θ verify*

$$\begin{aligned} \theta\lambda_1 + (1-\theta)\lambda_2 &= x_0 && \text{(8a)} \\ \theta\lambda_1^2 + (1-\theta)\lambda_2^2 - x_0^2 &= \kappa\sigma^2 && \text{(8b)} \end{aligned}$$

Equation (8a) equates the average of the two value probability law $(\lambda_1, \lambda_2, \theta)$ and the Gaussian distribution. Equation (8b) does the same with the variance. Equations (8a) and (8b) allow us to state that (assuming that $\theta \notin \{0,1\}$):

$$\begin{aligned} \lambda_1 &= x_0 + \frac{\sigma}{\sqrt{\theta}}\sqrt{1-\theta} && \text{(9a)} \\ \lambda_2 &= x_0 - \frac{\sigma}{\sqrt{\theta}}\left(\frac{1}{\sqrt{1-\theta}} - \sqrt{1-\theta}\right) && \text{(9b)} \end{aligned}$$

Since we are looking for three variables $(\lambda_1, \lambda_2, \theta)$, we need to impose a third equation. In our case, we added the following relation:

$$\lambda_1 = -\lambda_2$$

in order to capture the increasing and decreasing fluctuations of the oil price. A nonnegative value for λ implies an increase of the oil price, whereas a negative value means a decrease of the oil price. In our case, $\lambda_1 \geq 0$ and $\lambda_2 \leq 0$ correspond respectively to an increase and decrease of the Brent price.

The study of the Brent price between 1977 and 2009 gives the follwing values for λ_1, λ_2 and θ:

λ_1	0.1
λ_2	-0.1
θ	0.15

These values have been calculated for a five-year evolution of the Brent price. To summarize, the oil price evolves via the following formula:

$$p_{b+\kappa} = p_b^{1+\lambda_1} \quad \text{with probability } \theta \tag{10a}$$

$$p_{b+\kappa} = p_b^{1+\lambda_2} \quad \text{with probability } 1-\theta \tag{10b}$$

Relations (10a) and (10b) suggest that the oil price follows a Markov chain structure. This assumption has been verified and used in some statistical studies of the oil price such as (28). We belive that this representation of stochasticity is new and realistic in gas markets modeling. Indeed, in previous works, such as (9), (12), (29) and (18), no calibration of the demand's probability law is carried out.

The oil price evolution will create the scenario tree as follows: at each time-step the oil price can follow respectively equation (10a) or (10b) with probability θ and $1-\theta$. S-GaMMES's time scope is 2000-2035, with a time resolution of five years. In order to keep the model solvable in a reasonable time, we considered stochasticity only for the first five time-steps, until 2025. Starting from 2025, the oil price follows the trend forecast by the European Commission (10): an increase by 3.7% per year. The corresponding log-change percentage in that case is called μ. Figure 3 gives a schematic description of the scenario tree for the oil price and therefore for the demand function parameters. There are 31 nodes and seven time-steps (35 years). Node 0, which is the summit of the scenario tree corresponds to the 2000-2004 time period. Also note that stochasticity occurs starting from 2010. The figure gives the values of the different scenario nodes weights of the tree.

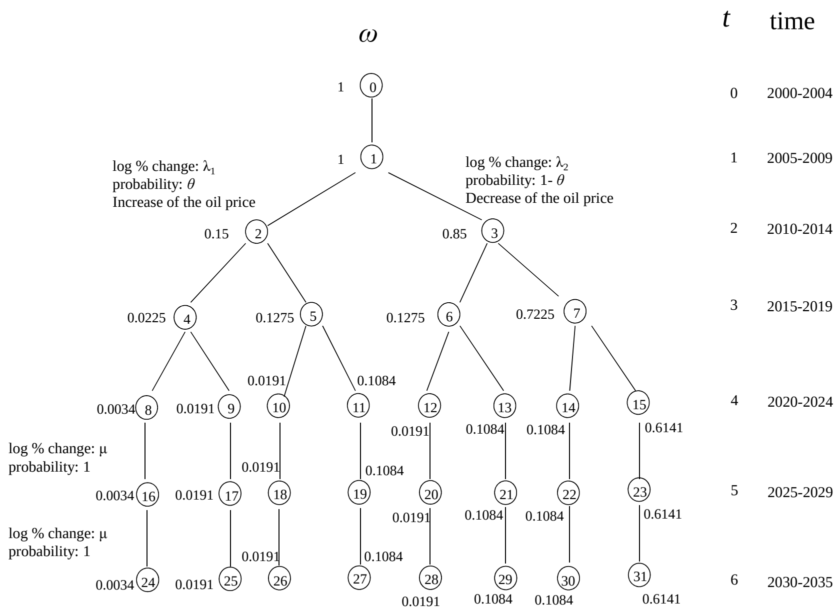

Figure 3:
The scenario tree weights.

Each node of the scenario tree represents the intersection of stochasticity with time. The first-stage variables are the ones decided by the players at nodes 0 and 1, which are deterministic. Once these variables have been chosen, they cannot be changed later, in the rest of the time periods (or nodes). Similarly, the decisions made at nodes 2 and 3 will influence the market outcome at all the forthcoming nodes $\omega \in \{4, 5, ...31\}$ especially the production, transport, and storage investments. More generally, an investment or a contractual decision made at node ω will remain unchanged and will influence the market structure at all the nodes ω' that follow ω in the tree.

3 The European natural gas markets model: numerical results

This section applies the model to the western European gas markets and presents the main paper's results.

3.1 The perimeter

The following array summarizes the main actors, production and storage sites and the seasons studied for the northwestern European natural gas market:

Producers		**Fields**		**Consuming markets**		**Independent traders**		**Storage sites**
Russia		Russia_f		France		France_{tr}		France_{st}
Algeria		Algeria_f		Germany		Germany_{tr}		Germany_{st}
Norway		Norway_f		The Netherlands		$\text{The Netherlands}_{tr}$		$\text{The Netherlands}_{st}$
The Nethherlands		NL_f		UK		UK_{tr}		UK_{st}
UK		UK_f		Belgium		Belgium_{tr}		Belgium_{st}

Seasons		**Time**		**Time step**		**# Scenario nodes**		**# Branches at each time step**
Peak/off-peak		$2000 - 2035$		5 *years*		31		2 or 1

We aggregate all the production fields of each producer into one production node. We assume that each consuming market is associated with one independent local trader (indexed by tr). As an example, France_{tr} would be GDF-SUEZ and Germany_{tr} would be E-On Ruhrgas. All the storage sites are also aggregated so that there is one storage node per consuming country. As for the transport, the different gas routes given in Figure 4 were considered.

The local production in the different consuming countries and the imports from non-represented producers, which are small, are exogenously taken into consideration.
We remind that the scenario tree representation we have used concerns the fluctuations of the oil price only.

The model has been solved, in its extensive form, using the solver PATH (11) from GAMS. Let us recall that in order to have an algorithm convergence in a reasonable time, we have used a five-year time-step resolution. We have chosen five years because it is the typical length of time needed to construct investments in production, infrastructure or storage.

3.2 The calibration

The calibration process has been carried out in order to best meet:

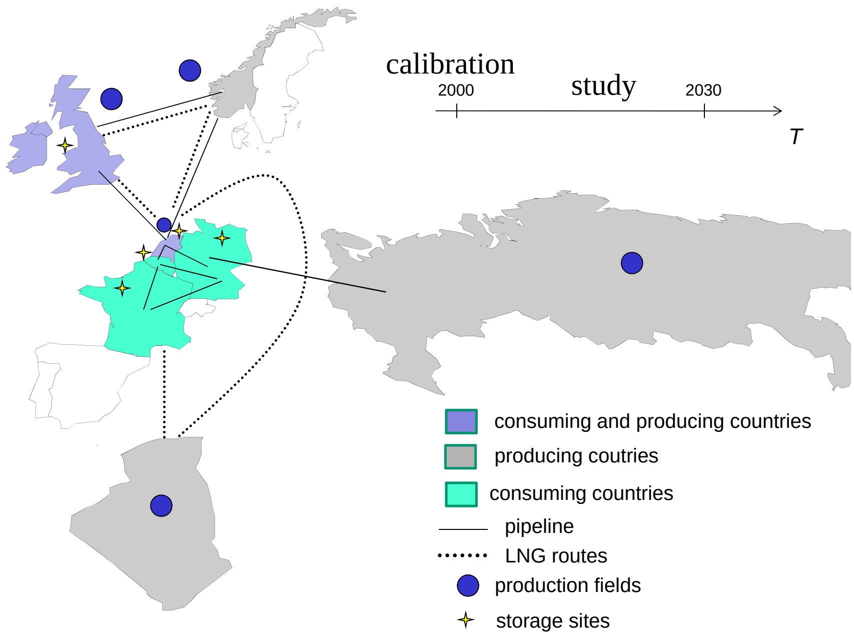

Figure 4:
The northwestern European natural gas routes, production, and storage sites.

- the global natural gas consumption and exchanges between producers and consumers,
- the industrial sector gas price and
- the volumes produced by each gas producer,

at scenario node 0. We recall that this node corresponds to the time period between 2000 and 2004 (the first time period). Therefore, there is no stochasticity associated with this node. The data for the market prices, consumed volumes, and imports is the publicly available set from IEA (19). The calibration tolerates a maximum error of 5% for the prices and consumed quantities and 10% for the imported/exported volumes. The tolerated error is higher for the exchanged volumes because they depend on the exports sent by the producers to all the targeted consumers, even those that are not in the scope of the model. As an example, the exported volumes from Russia to CIS (CEI) countries are exogenous to our model.

3.3 Results

In order to estimate the demand function parameters, our model requests exogenous inputs: the markets' global energy demand and coal price evolution (scenario provided by the European Commision (10)). We assume that the coal price remains constant and we have derived that the oil price follows a Markov chain regime. The annual gross consumption growth per year is given in the following chart (starting from 2000).

Annual growth	Total gross consumption (in % per year)
France	0.46
Germany	0.06
United Kingdom	0.02
Belgium	0.06
The Netherlands	0.11

Figure 5 gives the evolution of the consumption and prices in the demand markets between 2000 and 2035, for the different scenarios given in Figure 3.

We consider the different branches of the tree, indexed by their scenario node number (between 24 and 31) and draw the evolution of the consumption, with time, following the path in the tree that leads to the corresponding branch. It is important to remember that all the possible scenarios are solved simultaneously while respecting the non-anticipativity conditions. To analyze the actors' and markets' behavior, we also run S-GaMMES with two deterministic evolutions of the demand:

- A "High" demand case, where the oil price follows the path that leads to node 24. This corresponds to a deterministic increase of the oil price between 2000 and 2035 (a logarithmic change of $\lambda_1 \geq 0$).
- A "Low" demand case, where the oil price follows the path that leads to node 31. This corresponds to a deterministic decrease of the oil price between 2000 and 2035 (a logarithmic change of $\lambda_2 \leq 0$).

In the "High" and "Low" cases, it is assumed that the players know exactly, *a priori*, whether the oil prices are going to follow path 24 (constant and continuous increase of the oil price) or path 31 (constant and continuous decrease of the oil price).

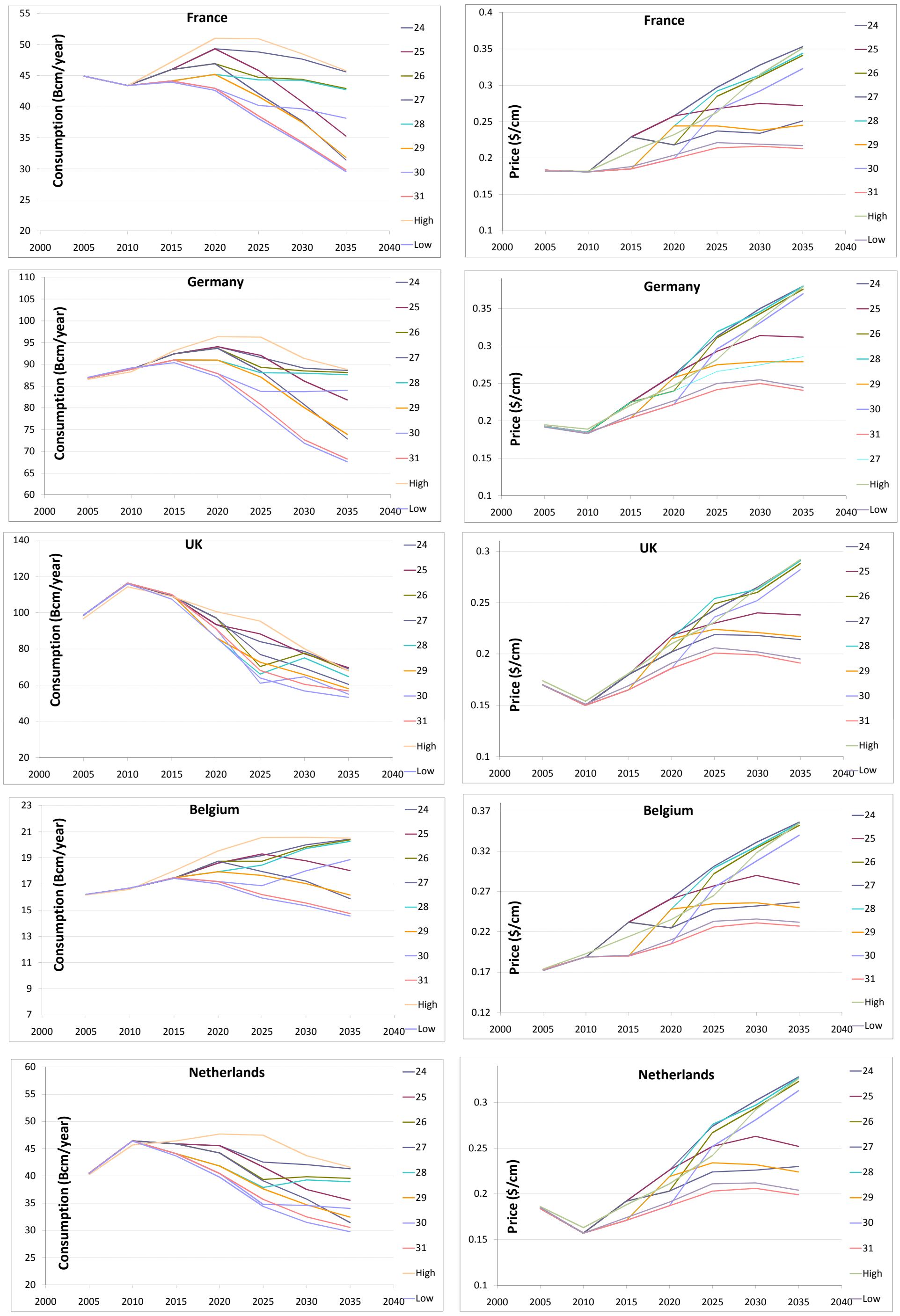

Figure 5:
Consumption and prices in the different scenarios.

One can notice that the evolution of the consumption and prices in all the scenarios is bounded by the deterministic "High" and "Low" scenarios. This result is intuitive because in both cases, it is assumed that the players have perfect foresight of the demand evolution. Besides, scenarios 24 and 31's results bind the other scenarios' consumption and prices evolution, because they correspond to a continuous and constant increase (scenario 24) or decrease (scenario 31) of the oil price.

The following table gives the consumption annual percentage growth (APG) mean value, between 2005 and 2035, in the European countries studied. The ratio between the standard deviation and the mean value of the consumption in 2035 (last period) is also given. The latter quantifies the impact of stochasticity on the spread of consumption with time. More particularly, we will define the spread by the following:

$$spread = \frac{\text{Standard deviation in 2035}}{\text{Mean value in 2035}} \tag{11}$$

The spread, a measure of the standard error, can be defined for all the parameters or market outcome that depend on the scenarios (consumption, prices, production, etc.). Intuitively, if the spread is high, this means that randomness may play an important role in the decisions made by the actors that influence the market outcome. This situation indicates that a stochastic model is more accurate to represent the market behavior as compared to a deterministic one. This notion will be detailed later while introducing and calculating the value of the stochastic solution.

Consumption	France	Germany	UK	Belgium	The Netherlands
(APG) mean value (%/year)	-1.15	-0.62	-1.76	-0.09	-0.81
Spread (in %)	11.1	8.5	4.9	10.4	6.5

One can notice that the spread has the highest value in France, which suggests that the fluctuations of the oil price influence the French gas consumption a great deal. On the contrary, the spread is relatively low in the United Kingdom. This country also has the highest decrease of consumption (in all scenarios). Indeed, the decrease of consumption is mainly due to the fact that the United Kingdom has low gas reserves (around 900 Bcm in 2000, (5)) and will have to rely on foreign (especially Russian and Algerian) supplies in the coming decades. Therefore, the evolution of the consumption in the UK is greatly dependent on the supply and less on the oil price fluctuations.

One can notice that the natural gas consumption is expected to decrease between 2000 and 2035, in most of the scenarios, even if the demand increases. This is mainly due to the fact that the initial natural gas reserves in Continental Europe are relatively low compared to the demand. Since we do not represent exploration activities (for shale gas for instance), the foreign dependence, especially toward Russia and Algeria, will grow with time, which will force the prices up mainly because of two reasons: first, the growing market power exerted by Russia and Algeria and second, the high transportation costs.

The following table gives the price annual percentage growth (APG) mean value and the spread, between 2005 and 2035, in the European countries studied.

Price	France	Germany	UK	Belgium	The Netherlands
(APG) mean value (%/year)	0.89	1.14	0.73	1.25	0.65
Spread (in %)	17.2	17.1	15.5	16.3	18.4

The spread is higher for the prices than for the consumption. This is intuitive because the gas prices are more correlated to the oil price as compared to the consumption. The Netherlands have the highest price spread in 2035.

Figure 6 shows the evolution of the production (dedicated to the consuming countries we studied) over time, in the different scenarios.

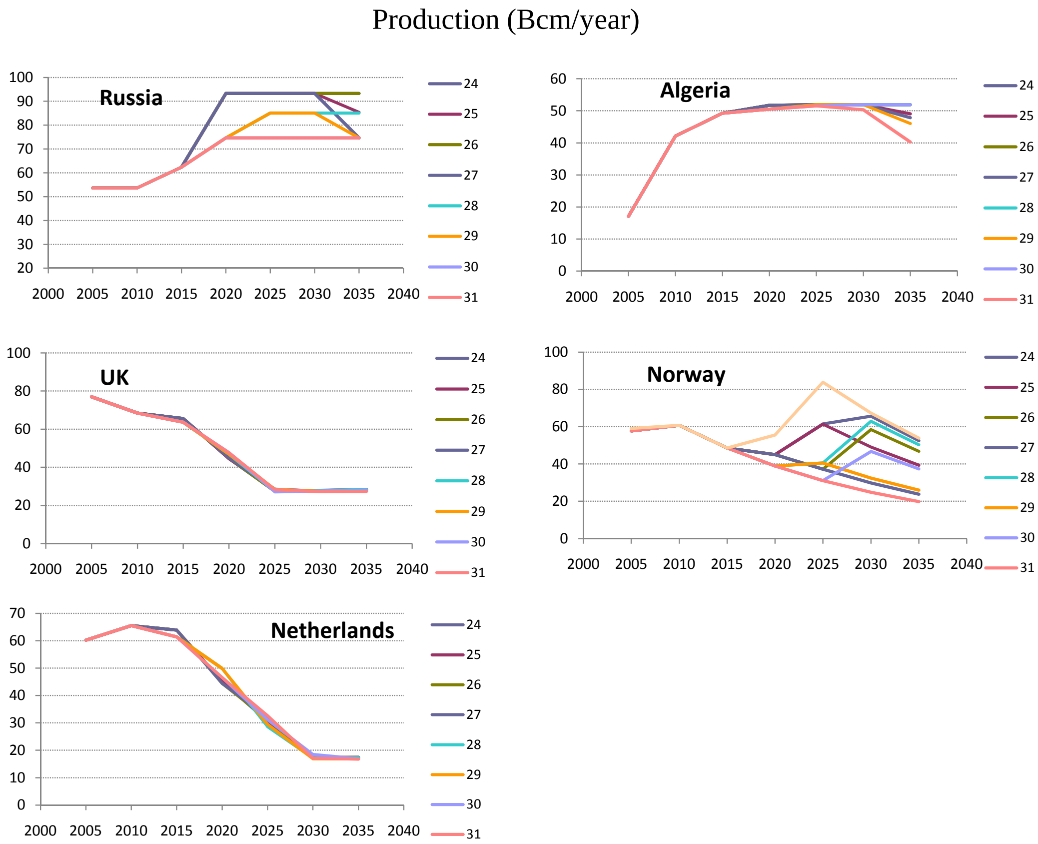

Figure 6:
Production in the different scenarios.

The Russian and Algerian shares in the European consumption is expected to grow, in all the possible scenarios. The Dutch and The UK production is expected to decrease with time, with a very small spread. This is mainly due to the limited reserves of gas initially present in these countries that reduces the correlation between the supply of these countries and the demand which is linked to the oil price that is stochastic. The Norwegian production varies a lot with the scenario and has a relatively high spread.

Now it may be interesting to analyze the impact of stochasticity on the long-term contracts. The following table gives the different long-term contracts' (LTC) volumes and prices between the producers and the independent traders. In order to compare with the deterministic version, we also report the LTC results in the High and Low cases [9].

Stochastic

Volume(Bcm/year)	France	Germany	UK	Belgium	The Netherlands	Total
Russia	0.84	32.26	0.00	0.00	0.00	33.10
Algeria	4.03	0.00	0.00	8.80	0.00	12.84
The Netherlands	0.00	0.00	0.00	9.42	5.91	15.33
Norway	0.00	0.00	0.00	12.16	0.00	12.16
UK	0.00	0.00	24.55	0.00	0.00	24.55
Total	4.87	32.26	24.55	30.38	5.91	97.98

High

Volume(Bcm/year)	France	Germany	UK	Belgium	The Netherlands	Total
Russia	0.31	33.61	0.00	0.00	0.00	33.92
Algeria	4.04	0.00	0.00	8.79	0.00	12.83
The Netherlands	0.00	0.00	0.00	9.23	5.84	15.06
Norway	1.11	0.00	2.88	12.44	0.00	16.43
UK	0.00	0.00	23.64	0.00	0.00	23.64
Total	5.47	33.61	26.53	30.45	5.84	101.89

Low

Volume(Bcm/year)	France	Germany	UK	Belgium	The Netherlands	Total
Russia	0.86	30.95	0.00	0.00	0.00	31.80
Algeria	3.93	0.00	0.00	8.88	0.00	12.81
The Netherlands	0.00	0.00	0.00	9.58	5.58	15.16
Norway	0.00	0.00	0.00	12.02	0.00	12.02
UK	0.00	0.00	24.89	0.00	0.00	24.89
Total	4.78	30.95	24.89	30.48	5.58	96.69

[9] nc denotes a no-contract situation.

Stochastic

Price($/cm)	France	Germany	UK	Belgium	The Netherlands
Russia	0.12	0.12	nc	nc	nc
Algeria	0.12	nc	nc	0.13	nc
The Netherlands	nc	nc	nc	0.13	0.14
Norway	nc	nc	nc	0.13	nc
UK	nc	nc	0.15	nc	nc

High

Price($/cm)	France	Germany	UK	Belgium	The Netherlands
Russia	0.13	0.13	nc	nc	nc
Algeria	0.13	nc	nc	0.14	nc
The Netherlands	nc	nc	nc	0.14	0.16
Norway	0.13	0.16	0.14	nc	nc
UK	nc	nc	0.16	nc	nc

Low

Price($/cm)	France	Germany	UK	Belgium	The Netherlands
Russia	0.12	0.12	nc	nc	nc
Algeria	0.12	nc	nc	0.12	nc
The Netherlands	nc	nc	nc	0.12	0.14
Norway	nc	nc	nc	0.12	nc
UK	nc	nc	0.15	nc	nc

One can notice that if a pair of producer-independent trader contract on the long-term, the corresponding LTC price is nonnegative. Besided, the spot prices in the consuming countries, reported in Figure 5 are in general higher than the LTC prices. One can also notice that the Belgian trader is the one that diversifies the most his gas supplies (three sources). This is due to its geographical location, which is close to three producing countries: Norway, The Netherlands and Algeria (we remind that the Algerian production node is directly linked to Belgium via a GNL route). A consuming country, which produces natural gas, such as the UK or The Netherlands, sees the corresponding independent trader contract on the long-term exclusively with the local producer. This is quite intuitive because of the geographical proximity. Besides, for a particular trader, the LTC price is the same with respect to all the possible supply sources. This suggests that the LTC prices are correlated to the spot prices. An independent trader may tolerate high supply marginal costs if his marginal revenue or the spot price in his spot market, which is the market where he has to support the least transportation costs, is high enough. Another explanation lies in the fact that at the equilibrium, no price arbitrage can be made between the producers to supply a midstreamer via LTCs.

A comparison between the Stochastic, High and Low cases shows that the LTC volumes contracted are, on average, 1.5% higher in the Stochastic case than in the Low case. On the contrary, LTC volumes are, on average, 4% lower in the Stochastic case than in the High case. There are even some contracts in the High case that do not exist in the Stochastic case: Norway-UK (2.9 Bcm/year) and Norway-France (1.1 Bcm/year). Regarding the prices, the results are similar: the Stochastic LTC prices are, on average 2.5% higher than in the Low case and the Stochastic LTC prices are, on average 9% lower than in the High case.

These results are quite intuitive: in the stochastic model, the strategic players need to hedge their decisions on the long-term, against the oil price fluctuations. In a High scenario perfect foresight, the demand increases constantly with time and the independent traders need to contract more important volumes in order to ensure a sufficient supply. In that situation, the spot price is expected to be relatively high (see Figure 5) and therefore the independent traders can support higher supply costs, which benefits to the producers. This explanation holds for the Stochastic-Low cases LTC comparison.

We believe that these results hold interesting policy implications in the upstream gas trade, because they show that the contract structure depends on the percieved uncertain evolution of the demand by the producers and midstreamers.

3.4 The value of the stochastic solution

Now we define a measure that allows us to quantify the utility to introduce stochasticity in the S-GaMMES model. Following (29), we adapt the concept of the value of the stochastic solution, introduced by (4) to analyze the performance of stochastic programing, to the context of imperfect competition and equilibrium problems.

For that purpose, we will compare the Stochastic Model (SM) and the Mean Value Model (MVM) results, which will be defined later. In our representation, at each time period, the oil price is modeled as a random variable whose mean value will be denoted by:

$$p_t = \langle p_t^{\omega} \rangle_{\omega} \tag{12}$$

The mean value is calculated by considering all the scenarios ω that correspond to time-step t. The Mean Value Model is a deterministic model where, at each time step, we approximate the oil price by its mean value p_t. Figure 7 is a schematic description of both models.

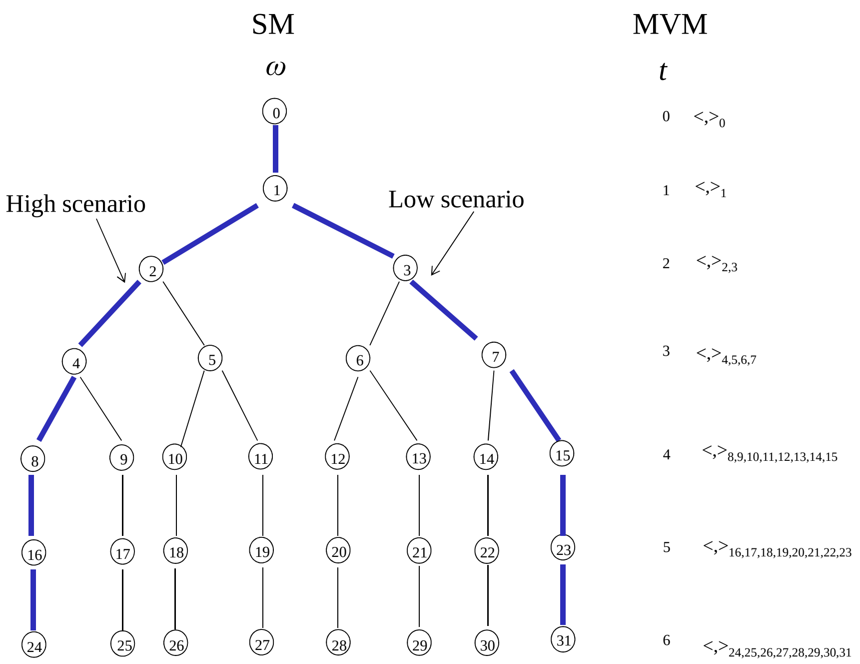

Figure 7:
The stochastic (SM), Mean Value (MVM), High (HM) and Low (LM) Models.

We will compare the different players' utilities in the Stochastic and Mean Value cases. We will also compare the Stochastic, the High demand and Low demand cases utilities. For a particular player, we will define:

- The SM utility Π_{SM} by the expected value of its global utility over the possible scenarios, in the SM output.
- The MVM utility Π_{MVM} by the value of its global utility over time, in the MVM output.
- The HM utility Π_{HM} by the value of its global utility over time, in the High case output.
- The LM utility Π_{LM} by the value of its global utility over time, in the Low case output.
- The value of the stochastic solution defined by:

$$VSS = \Pi_{SM} - \Pi_{MVM} \tag{13}$$

- The loss of the stochastic solution defined by:

$$LSS = \Pi_{SM} - \Pi_{HM} \tag{14}$$

- The gain of the stochastic solution defined by:

$$GSS = \Pi_{SM} - \Pi_{LM} \tag{15}$$

Note that the Mean Value Model, the High demand and Low demand cases are deterministic situations, where the players have a perfect foresight of the future. The VSS is a means to quantify the importance of using stochasticity in the model. The GSS measures the gain obtained by the players when taking into consideration stochasticity, as compared to a deterministic low demand regime. To the contrary, the LSS measures the loss supported by the players when taking into consideration stochasticity, as compared to a deterministic high demand regime. In linear and non-linear stochastic programing, Birge and Louveaux (4) demonstrated that the VSS is nonnegative. However, this result may not hold for MCPs or Equilibrium Stochastic Problems. Indeed, since we do not deal with a unique objective function to optimize, but rather with multiple correlated ones, it is not straightforward that each player would benefit from taking care of stochasticity. The same conclusions hold for the GSS and LSS. Indeed, in stochastic programing, it is intuitive that the GSS is nonnegative whereas the LSS is negative. However this may not be true when dealing with MCP formulations.

The following table gives the VSS, GSS and LSS for the producers [10]. A producer's utility is his profit.

	VSS (*10^9 \$)	VSS (%)	GSS (%)	LSS (%)
Russia	9.23	9.23	19.45	-19.85
Algeria	0.91	1.43	7.23	-17.32
The Netherlands	-0.87	-0.58	0.75	-4.69
Norway	-0.29	-0.29	2.4	-14.66
UK	-1.18	-0.72	0.8	-5.42
Total	7.80	1.35	4.80	-10.79

The average VSS for the producers is 1.35%, which is nonnegative. This means that on average, the producers benefit from the use of stochasticity in their optimization programs. There are some cases where the VSS is negative (The Netherlands, Norway and The UK). However, the corresponding values are relatively small. Russia is the producer that benefits the most from the use of stochasticity, with a VSS of 9%. Regarding the GSS and LSS, their average values are 4.8% and -10.8% respectively. All the producers have a nonnegative GSS and negative LSS.

The following table gives the VSS, GSS and LSS for the independent traders. An independent trader's utility is his profit.

	VSS (*10^9 \$)	VSS (%)	GSS (%)	LSS (%)
France	-0.03	-0.54	3.89	-27.08
Germany	0.15	0.37	4.09	-13.40
UK	0.17	1.28	1.01	-18.17
Belgium	-0.15	-0.51	1.24	-6.81
The Netherlands	0.55	14.29	12.08	2.55
Total	0.69	0.73	3.05	-12.17

The average VSS for the traders is 0.73%, which is nonnegative. This means that on average, the independent traders benefit from the use of stochasticity in their optimization programs.

[10] The VSS in % is defined by $\frac{\Pi_{SM}-\Pi_{MVM}}{\Pi_{SM}}$. The definition is similar for the GSS and LSS in %.

Nevertheless, there are some cases where the VSS is negative. The Dutch trader is the one that benefits the most from the use of stochasticity, with a VSS of 14%. The average GSS is 3.05% and the average LSS is -12.17%. The Dutch trader has a positive LSS. However, the value is relatively small compared to the GSS or the VSS.

The previous results concerned the strategic players, who directly take into consideration stochasticity in their profits. Now it may be interesting to measure the VSS for the non-strategic players.

The following table gives the VSS, GSS and LSS for the consumers. The consumers' utility is their surplus: if the inverse demand function is $p(q)$ and the consumed quantity is Q, then the utility is defined by $\int_0^Q (p(q) - p(Q))\, q\ dq$.

	VSS (*10^9 \$)	VSS (%)	GSS (%)	LSS (%)
France	0.76	0.28	2.43	-16.52
Germany	1.86	0.46	2.49	-9.28
UK	3.23	2.54	2.71	-2.93
Belgium	0.33	0.42	2.39	-11.1
The Netherlands	1.37	1.26	2.9	-12.5
Total	7.55	0.76	2.54	-10.88

The average VSS for the consuming countries is 0.76%, which is nonnegative. This means that in general, the consumers benefit from the use of stochasticity. Note that this result is not intuitive because the consumers' surplus is not taken into account in the producers or the traders payoff. However, this can be explained by the fact that in S-GaMMES, the strategic players have a better representation of the demand fluctuations that directly influences the consumers' surplus. The United Kingdom is the country that benefits the most from the use of stochasticity, with a VSS of 2.5%. The average GSS and LSS values are respectively 2.5% and -10.88%

The following table gives the VSS, GSS and LSS for the pipeline (TSO) and storage operators (SSO). The utility is the opposite of the cost they minimize. The VSS and GSS are also nonnegative, whereas the LSS is negative.

	VSS ($*10^9$ \$)	VSS (%)	GSS(%)	LSS(%)
TSO	5.69	8.33	0.67	-38.67
SSO	0	0	0	0

In conclusion, the use of the Value of the Stochastic Solution allows us to quantify the gain earned by the players (strategic and non-strategic) when considering stochasticity in their decisions. When calculated in S-GaMMES, the results suggest that, on average, all the strategic and non-strategic players benefit from the use of stochasticity. However, since we are dealing with an Equilibrium Stochastic Problem, there are some players who may suffer from that situation, which may lead to their utility becoming smaller.

4 Conclusion

S-GaMMES is relatively well suited to describe the European gas trade, which is still mainly dominated by long-term contracts (LTC) in the upstream and where the LTCs are oil price-indexed. The inverse demand function explicitly takes into consideration the possible substitution between consumption of natural gas and the competing fuels.

One of the S-GaMMES model's key features is that it captures the markets' long-term aspects in an endogenous way, for both long-term contract prices and volumes. It can be proved that long-term contracts' prices, or LTC prices, are smaller than the spot market prices. Indeed, since long-term contracts are the only means for the independent traders to obtain gas, LTC prices are to be considered as supply costs for them. Besides, they make a profit by selling natural gas directly to the consumers, in the spot markets. Therefore, if the traders have an incentive to sell gas to the consumers, their revenue must be greater than their costs and consequently, spot prices should be, on average, greater than LTC prices.

We apply our model to the northwestern European gas trade to forecast between 2000 and 2035 consumption, prices, production, and LTC prices and volumes, in the different possible scenarios allowed by the scenario tree. We compare in particular LTC prices in the stochastic versus the deterministic situations, in order to understand how the producers hedge their investment-related risk when dealing with an uncertain demand. Furthermore, we define and calculate the value (as well as the gain and loss) of the stochastic solution adapted to the model. This allows to quantify the importance and usefulness of taking into consideration the demand randomness, as compared to deterministic evolutions.

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1 Notation

Exogenous factors

P	set of producers-dedicated traders
I	set of independent traders
D	set of gas consuming countries in the downstream market (no distinction between the sectors) $D \subset N$
T	time $T = \{0, 1, 2, ..., Num\}$
M	set of seasons. Off-peak (low-consumption) and peak (high-consumption) regimes
F	set of all the gas production fields. $F \subset N$
N	set of the nodes
S	set of the storage sites $S \subset N$
A	set of the arcs (topology)
Ω	set of scenario nodes
Ω_l	set of the tree leaves $\Omega_l \subset \Omega$
Rf_f	field f's total gas resources (endowment)
Kf_f	field f's initial capacity of production, year 0
Lf_f	production node f's maximum increase of the production capacity (in %)
Ic_s	injection marginal cost at storage site s (constant)
Wc_s	withdrawal marginal cost at storage site s (constant)
Rc_s	reservation marginal cost at storage site s (constant)
Ls_s	storage node s's maximum increase of the storage capacity (in %)
Pc_f	production cost function, field f
Tc_a	transport marginal cost through arc a (constant)
Tk_a	pipeline initial capacity through arc a, year 0
Ks_s	initial storage capacity at site s, year 0
Is_s	investment marginal costs in storage (constant)
Ip_f	investment marginal costs in production (constant)
Ik_a	investment marginal costs in pipeline capacity through arc a (constant)
La_a	arc a's maximum increase of the transport capacity (in %)
O	incidence matrix $\in \mathrm{M}_{F\times P}$. $O_{fp} = 1$ if and only if producer p owns field f
B	incidence matrix $\in \mathrm{M}_{I\times D}$. $B_{id} = 1$ if and only if trader i is located at the consumption node d
$M1$	incidence matrix $\in \mathrm{M}_{F\times N}$. $M1_{fn} = 1$ if and only if node n has field f
$M2$	incidence matrix $\in \mathrm{M}_{I\times N}$. $M2_{in} = 1$ if and only if trader i is located at node n
$M3$	incidence matrix $\in \mathrm{M}_{D\times N}$. $M3_{dn} = 1$ if and only if node n has market d
$M4$	incidence matrix $\in \mathrm{M}_{S\times N}$. $M4_{sn} = 1$ if and only if node n has storage site s
$M5$	incidence matrix $\in \mathrm{M}_{A\times N}$. $M5_{an} = 1$ if and only if arc a starts at node n
$M6$	incidence matrix $\in \mathrm{M}_{A\times N}$. $M6_{an} = 1$ if and only if arc a ends at node n
(ω)	probability of occurence of scenario node ω
$t(\omega)$	time associated with scenario node ω
H	maximum value for the quantities produced and consumed
δ^{ω}_{md}	an inverse demand function parameter
β^{ω}_{md}	an inverse demand function parameter
γ^{ω}_{md}	an inverse demand function parameter
pc^{ω}_{md}	an inverse demand function parameter
fl_f	field f's flexibility: the maximum spread between production during off-peak and peak seasons
min_{pi}	percentage of the minimum quantity that has to be exchanged on the LTC trade between i and p
δ	discount factor
$delay_{s,i,p}$	period of time necessary to undertake technical investments
$loss_a$	loss factor through arc a
dep_f	depreciation factor of the production capacity at field f

Endogenous variables

x^{ω}_{mfpd}	quantity (Bcm) of gas produced by p from field f for the end-use market d, scenario node ω, season m
zp^{ω}_{mfpi}	quantity (Bcm) of gas produced by p from field f dedicated to the long-term contract with trader i, scenario node ω, season m
zi^{ω}_{mpi}	quantity (Bcm) of gas bought by trader i from producer p with a long-term contract scenario node ω, season m
up_{pi}	quantity (Bcm) of gas sold by producer p to trader i with a long-term contract, each year
ui_{pi}	quantity (Bcm) of gas bought by trader i from producer p on the long-term contract, each year
y^{ω}_{mid}	quantity (Bcm) of gas sold by i to the market d, scenario node ω, season m
ip^{ω}_{fp}	producer p's increase of field f's production capacity (in Bcm/time unit), due to investments in production, scenario node ω
q^{ω}_{mfp}	production (in Bcm) of producer p from field f, scenario node ω, season m
p^{ω}_{md}	market d's gas price (in \$/cm), result of the Cournot competition between all the traders, scenario node ω, season m
η_{pi}	long-term contract price (in \$/cm) contracted between producer p and trader i
r^{ω}_{is}	amount of storage capacity (in Bcm) reserved by trader i at site s, scenario node ω
in^{ω}_{is}	volume (in Bcm) injected by trader i at site s, scenario node ω
is^{ω}_{s}	increase of storage capacity (in Bcm/time unit) at site s, scenario node ω, due to the storage operator investments
ik^{ω}_{a}	increase of the pipeline capacity (in Bcm/time unit) through arc a, scenario node ω, due to the TSO investments
$fp^{\omega}_{m,p,a}$	gas quantity (in Bcm) that flows through arc a from producer p scenario node ω, season m
$fi^{\omega}_{m,i,a}$	gas quantity (in Bcm) that flows through arc a from trader i scenario node ω, season m
$\tau^{\omega}_{m,a}$	the dual variable associated with arc a capacity constraint (in Bcm/season) scenario node ω, season m It represents the congestion transportation cost over arc a

The units for quantities are in toe (i.e., Ton Oil Equivalent) or Bcm (i.e., 10^9 cubic meters) and units for prices are in \$/toe or \$/cm. The indices p, d, i, f, n, s, a, m, ω and t are such that $p \in P$, $d \in D$, $i \in I$ $f \in F$, $n \in N$, $s \in S$, $a \in A$, $m \in M$, $\omega \in \Omega$ and $t \in T$. In the remainder of the paper and according to the context, a node can either represent a geographical location (of a production field, a consumption market or a storage site) or a location in the scenario tree. The long-term contract between producer p and trader i fixes both a unit selling price and an amount to be purchased by the independent trader i each year from producer p. Both price and quantity will be specified endogenously by the model. Matrix O is such that $O_{fp} = 1$ if producer p owns field f and $O_{fp} = 0$ otherwise.

2 Agents' decisions

2.1 Agents' behavior

This section presents the optimization problems of all the supply chain players.[1]

Producer p's maximization program is given below. The corresponding decision variables are zp^{ω}_{mfpi}, x^{ω}_{mfpd}, ip^{ω}_{fp}, q^{ω}_{mfp} and up_{pi}.

[1]Note that the dual variables are written in parentheses next to their associated constraints.

$$
\begin{aligned}
\text{Max} & \sum_{\omega,m,f,i} \pi(\omega)\delta^{t(\omega)}(\eta_{pi})(zp^{\omega}_{mfpi}) \\
& + \sum_{\omega,m,f,d} \pi(\omega)\delta^{t(\omega)} \left(p^{\omega}_{md}(x^{\omega}_{mfpd} + \overline{x^{\omega}_{mfpd}})\right) x^{\omega}_{mfpd} \\
& - \sum_{\omega,f} \pi(\omega)\delta^{t(\omega)} Pc_f \left(\sum_{\omega' \leq \omega} \sum_{m} q^{\omega'}_{mfp}, Rf_f \right) \\
& + \sum_{\omega,f} \pi(\omega)\delta^{t(\omega)} Pc_f \left(\sum_{\omega' < \omega} \sum_{m} q^{\omega'}_{mfp}, Rf_f \right) \\
& - \sum_{\omega,f} \pi(\omega)\delta^{t(\omega)} Ip_f(ip^{\omega}_{fp}) \\
& - \sum_{\omega,m,a} \pi(\omega)\delta^{t(\omega)}((Tc_a + \tau^{\omega}_{m,a}) fp^{\omega}_{m,p,a})
\end{aligned}
$$

such that:

$$
\begin{array}{rlcll}
\forall \omega,\ f, & \displaystyle\sum_{p} \sum_{\omega' \leq \omega} \sum_{m} q^{\omega}_{mfp} - Rf_f & \leq 0 & (\phi^{\omega}_{f}) & \text{(1a)} \\
\forall \omega,\ f,\ m, & \displaystyle\sum_{p} q^{\omega}_{mfp} - Kf_f(1-dep_f)^{t(\omega)} & & & \\
& \displaystyle - \sum_{p} \sum_{\omega' < \omega} ip^{\omega'}_{fp}(1-dep_f)^{t(\omega)-t(\omega')} & \leq 0 & (\chi^{\omega}_{mf}) & \text{(1b)} \\
\forall \omega,\ m,\ f, & \displaystyle - q^{\omega}_{mfp} + \left(\sum_{i} zp^{\omega}_{mfpi} + \sum_{d} x^{\omega}_{mfpd} \right) & \leq 0 & (\gamma^{\omega}_{mfp}) & \text{(1c)} \\
\forall \omega,\ f,\ p, & \displaystyle\sum_{m} ((-1)^m q^{\omega}_{mfp}) - fl_f & \leq 0 & (\vartheta 1^{\omega}_{fp}) & \text{(1d)} \\
\forall \omega,\ f,\ p, & \displaystyle - \sum_{m} ((-1)^m q^{\omega}_{mfp}) - fl_f & \leq 0 & (\vartheta 2^{\omega}_{fp}) & \text{(1e)} \\
\forall \omega,\ f,\ d,\ m, & x^{\omega}_{mfpd} - O_{fp}H & \leq 0 & (\epsilon 1^{\omega}_{mfpd}) & \text{(1f)} \\
\forall \omega,\ f,\ i,\ m, & zp^{\omega}_{mfpi} - O_{fp}H & \leq 0 & (\epsilon 2^{\omega}_{mfpi}) & \text{(1g)} \\
\forall \omega,\ f,\ m, & q^{\omega}_{mfp} - O_{fp}H & \leq 0 & (\epsilon 3^{\omega}_{mfp}) & \text{(1h)} \\
\forall \omega,\ f, & ip^{\omega}_{fp} - O_{fp}H & \leq 0 & (\epsilon 4^{\omega}_{fp}) & \text{(1i)} \\
\forall \omega,\ f, & \displaystyle\sum_{p} ip^{\omega}_{fp} - Lf_f Kf_f (1-dep_f)^{t(\omega)} & & & \\
& \displaystyle - Lf_f \sum_{p} \sum_{\omega' < \omega} ip^{\omega'}_{fp}(1-dep_f)^{t(\omega)-t(\omega')} & \leq 0 & (\iota p^{\omega}_{f}) & \text{(1j)} \\
\forall \omega,\ m,\ n, & \displaystyle\sum_{a} M6_{an} fp^{\omega}_{m,p,a}(1-loss_a) - \sum_{a} M5_{an} fp^{\omega}_{m,p,a} & & & \\
& \displaystyle + \sum_{f} q^{\omega}_{mpf} M1_{fn} - \sum_{d} \sum_{f} x^{\omega}_{mfpd} M3_{dn} & & & \\
& \displaystyle - \sum_{i} \sum_{f} zp^{\omega}_{mfpi} M2_{in} & = 0 & (\alpha p^{\omega}_{m,p,n}) & \text{(1k)} \\
\forall \omega,\ i, & \displaystyle up_{pi} - \sum_{f,m} zp^{\omega}_{mfpi} & = 0 & (\eta p^{\omega}_{pi}) & \text{(1l)} \\
\forall\ p,\ i, & ui_{pi} - up_{pi} & = 0 & (\eta_{pi}) & \text{(1m)} \\
\forall \omega,\ m,\ d,\ i,\ f, & zp^{\omega}_{mfpi},\ x^{\omega}_{mfpd},\ ip^{\omega}_{fp},\ q^{\omega\,4}_{mfp},\ up_{pi} & \geq 0 & &
\end{array}
$$

We denote by $\overline{x^{\omega}_{mfpd}}$ the total amount of gas brought at node ω, season m to the market d by all the players different from producer p.
The term $\sum_{\omega,m,f,i}\pi(\omega)\delta^{t(\omega)}(\eta_{pi})(zp^{\omega}_{mfpi})+\sum_{\omega,m,f,d}\pi(\omega)\delta^{t(\omega)}\left(p^{\omega}_{md}(x^{\omega}_{mfpd}+\overline{x^{\omega}_{mfpd}})\right)x^{\omega}_{mfpd}$ is the revenue, which is obtained from the sales from the long-term contract sales to the independent traders or directly from the retail markets.
The term $\sum_{\omega,f}\pi(\omega)\delta^{t(\omega)}Pc_f\left(\sum_{\omega'\leq\omega}\sum_m q^{\omega'}_{mfp}, Rf_f\right)-\sum_{\omega,f}\pi(\omega)\delta^{t(\omega)}Pc_f\left(\sum_{\omega'<\omega}\sum_m q^{\omega'}_{mfp}, Rf_f\right)$ is the actualized production cost. This term's explanation is as follows:
The production cost (at field f) Pc_f depends on two variables, the total quantity produced, which will be denoted q and the natural gas resources Rf_f. The Golombek production cost function (see **?** and **?**) we used is as follows:

$$\forall q\in[0,Rf_f),\ Pc_f(q,Rf_f)=a_f q+b_f\frac{q^2}{2}-\ Rf_f c_f\left(\frac{Rf_f-q}{Rf_f}\ln\left(\frac{Rf_f-q}{Rf_f}\right)+\frac{q}{Rf_f}\right) \tag{2}$$

or if written for the marginal production cost

$$\forall q\in[0,Rf_f),\ \frac{\mathrm{d}Pc_f}{\mathrm{d}q}=a_f+b_f q\ +\ c_f\ln\left(\frac{Rf_f-q}{Rf_f}\right) \tag{3}$$

In our model, the production cost function is dynamic. The gas volume available to be extracted is dynamically reduced at each period, taking into account the exhaustivity of the resource. If at time-step 1, the production is $q1$ and at time-step 2 $q2$, the total cost is hence:

$$cost=Pc_f(q1,RES_f)+\delta(Pc_f(q1+q2,RES_f)-Pc_f(q1,RES_f))$$

Thus, to estimate the cost at scenario node ω, we need to calculate the production cost of the sum over all the extracted volumes until node ω and subtract the cost we have cummulated at all the strict predecessor nodes to ω.
The term $\sum_{\omega,f}\pi(\omega)\delta^{t(\omega)}Ip_f(ip^{\omega}_{fp})$ is the investment cost in production at the different production fields.
The term $\sum_{\omega,m,a}\pi(\omega)\delta^{t(\omega)}((Tc_a+\tau^{\omega}_{m,a})fp^{\omega}_{m,p,a})$ is the transport and congestion costs charged by the pipeline operator to producer p. The dual variable $\tau^{\omega}_{m,ar}$ is associated with the pipeline capacity constraint through the arc a. It represents the congestion price on the corresponding pipeline (see the transport operator optimization problem for a more detailed explanation).
The explanation of the constraints is straightforward:

The constraint (1a) bounds each field's production by its reserves.

The constraint (12b) bounds the seasonal quantities produced by each field's production capacity, taking explicitly into account the different dynamic investments, that decrease with time because of the production depreciation factor. To take into consideration the investment delays, we account only for the invested capacities at the strict predecessor nodes. This corresponds to a five-year investment delay (the time-step of the model).

The constraint (1c) states that the total production must be greater than the sales (to the long-term and spot markets).

The constraints (1d) and (1e) can be rewritten as $\forall\omega,\ f,\ p,\ |\sum_m((-1)^m q^{\omega}_{mfp})|\leq fl_f$. This fixes a maximum spread between the off-peak/peak production at each field. $(-1)^m$ is equal to 1 in the off-peak season and -1 in the peak season.

The constraint (1k) is a market-clearing condition at each node, regarding the flows from producer p depending on whether this node is a production field, an independent trader location or a demand market.

The constraint (1j) bounds the capacity expansion of each production node f: each year, the investment decided to increase the production capacity is less than $100\times Lf_f$ percent the

installed capacity at that year. A historical study of the capacity expansion of some production nodes allowed us to calibrate the value of Lf_f: $Lf_f = 0.20$.
The constraint (1l) equates the sales of producer p for the long-term contracts to the contracted volume up_{pi}, each scenario node.

The constraint (1m) describes the following: For each pair of producer/independent trader (p, i), the gas quantity sold by p in the long-term contract market must be equal to the gas quantity purchased by i. Therefore, this is a supply/demand equation in the long-term contracts market. The associated dual variable η_{pi} is the corresponding contract unit selling/purchase price, because we do not assume the existence of market power in the long-term contract trade. Using this technique, it is possible to make the long-term contract prices and volumes endogenous to the description so that they become an output of the model.

The constraint (and the similar other ones) (1f) allows producer p to use only the fields he owns (for production, investments, sales, etc.). We recall that the incidence matrix O is such as $O_{fp} = 1$ if and only if producer p owns field f, otherwise, $O_{fp} = 0$.

Independent trader i's maximization program is given below. The corresponding decision variables are zi^{ω}_{mpi}, y^{ω}_{mid}, r^{ω}_{is}, in^{ω}_{is} and ui_{pi}.

$$
\begin{aligned}
\text{Max} & \sum_{\omega,m,d} \pi(\omega)\delta^{t(\omega)} \left(p^{\omega}_{md}(y^{\omega}_{mid} + \overline{y^{\omega}_{mid}})y^{\omega}_{mid}\right) \\
& - \sum_{\omega,p,m} \pi(\omega)\delta^{t(\omega)} \left(\eta_{pi} zi^{\omega}_{mpi}\right) \\
& - \sum_{\omega,s} \pi(\omega)\delta^{t(\omega)} \left(Rc_s(r^{\omega}_{is})\right) \\
& - \sum_{\omega,s} \pi(\omega)\delta^{t(\omega)} \left((Ic_s + Wc_s) in^{\omega}_{is}\right) \\
& - \sum_{\omega,m,a} \pi(\omega)\delta^{t(\omega)} \left(Tc_a + \tau^{\omega}_{m,a}\right) fi^{\omega}_{m,i,a}
\end{aligned}
$$

such that:

$$
\begin{aligned}
&\forall \omega,\ m, && \sum_p zi^{\omega}_{mfpi} - \left(\sum_d y^{\omega}_{mid} + (-1)^m \sum_s in^{\omega}_{is}\right) && = 0 && (\psi^{\omega}_{mi}) && \text{(4a)} \\
&\forall \omega,\ s, && in^{\omega}_{is} - r^{\omega}_{is} && \le 0 && (\mu^{\omega}_{is}) && \text{(4b)} \\
&\forall \omega,\ m,\ n, && \sum_a M6_{an} fi^{\omega}_{m,i,a}(1 - loss_a) - \sum_a M5_{an} fi^{\omega}_{m,i,a} && && && \\
& && - \sum_d y^{\omega}_{mfid} M3_{dn} + \sum_p zi^{\omega}_{mpi} M2_{in} && && && \\
& && - (-1)^m \sum_s M4_{sn} \left(\sum_i ij^{\omega}_{is}\right) && = 0 && (\alpha i^{\omega}_{m,p,n}) && \text{(4c)} \\
&\forall \omega,\ p, && ui_{pi} - \sum_m zi^{\omega}_{mpi} && = 0 && (\eta i^{\omega}_{pi}) && \text{(4d)} \\
&\forall\ p,\ i, && ui_{pi} - up_{pi} && = 0 && (\eta_{pi}) && \text{(4e)} \\
&\forall \omega,\ m,\ p,\ i, && - zi^{\omega}_{mpi} + min_{pi} \sum_m zi^{\omega}_{mpi} && \le 0 && (\upsilon^{\omega}_{mpi}) && \text{(4f)} \\
&\forall \omega,\ m,\ s,\ d, && zi^{\omega}_{mpi},\ y^{\omega}_{mid},\ r^{\omega}_{is},\ in^{\omega}_{is},\ ui_{pi} && \ge 0 && &&
\end{aligned}
$$

The term $\sum_{\omega,m,d}\pi(\omega)\delta^{t(\omega)}\left(p^{\omega}_{md}(y^{\omega}_{mid}+\overline{y^{\omega}_{mid}})y^{\omega}_{mid}\right)-\sum_{\omega,p,m}\pi(\omega)\delta^{t(\omega)}\left(\eta_{pi}zi^{\omega}_{mpi}\right)$ is the net profit.

The term $\sum_{\omega,s}\pi(\omega)\delta^{t(\omega)}\left(Rc_s(r^{\omega}_{is})\right)$ is the storage capacity reservation cost.

The term $\sum_{\omega,s}\pi(\omega)\delta^{t(\omega)}\left((Ic_s+Wc_s)in^{\omega}_{is}\right)$ are the storage/withdrawal costs[2].

The term $\sum_{\omega,m,a}\pi(\omega)\delta^{t(\omega)}\left(Tc_a+\tau^{\omega}_{m,a}\right)fi^{\omega}_{m,i,a}$ is the transport and congestion costs charged by the pipeline operator to the independent trader i. As for the feasibility set, it is also easy to specify :

The constraint (4a) is a gas quantity balance for each trader. The term $(-1)^m$ is equal to 1 in the off-peak season and -1 otherwise. An implicit assumption we use in the description is that all the storage sites must be "empty" (regardless of the working gas quantitities) at the end of each year.

The equation (4b) implies that each independent trader has to pay for a storage reservation quantity, each year and at each storage site s, to be able to store his gas.

The constraint (4d) forces each trader to purchase the same quantity, in long-term contracts, from each producer and scenario node.

The constraint (4e) is similar to the constraint (1m) of the producers' optimization program. For each pair of producer/independent trader (p,i), the gas quantity sold by p in the long-term contract market must be equal to the gas quantity purchased by i. Therefore, this is a supply/demand equation in the long-term contracts market. The associated dual variable η_{pi} is the corresponding contract unit selling/purchase price, because we do not assume the existence of market power in the long-term contract trade. Using this technique, it is possible to make the long-term contract prices and volumes endogenous to the description so that they become an output of the model.

The constraint (4f) fixes a minimum percentage of the contracted volume, per time unit, min_{pi} that has to be exchanged between p and i each season of each scenario node. Obviously, this constraint is expected to be more saturated in the summer when there is little need for the traders to have an important amount of gas supply.

On the transportation side of our model, we will assume that the producers pay the transport costs to bring natural gas from the production fields to the independent traders' locations and the end-use markets. The traders support the transport costs to store/withdraw gas or bring it to the end-users for their sales. All the distribution costs are implicitly included in the transportation costs we use.

[2]There are no storage losses in the model. They can easily be taken into account by increasing the transportation losses of the arcs that start at the storage nodes.

The pipeline operator optimization (cost minimization) program is given below. The corresponding decision variables are $fp^{\omega}_{m,p,a}$, $fi^{\omega}_{m,i,a}$ and ik^{ω}_{a}.

$$
\begin{aligned}
\text{Min} \sum_{\omega,m,a} & \pi(\omega)\delta^{t(\omega)} \left(Tc_a + \tau^{\omega}_{m,a}\right) \sum_{p} fp^{\omega}_{m,p,a} \\
& + \sum_{\omega,m,a} \pi(\omega)\delta^{t(\omega)} \left(Tc_a + \tau^{\omega}_{m,a}\right) \sum_{i} fi^{\omega}_{m,i,a} \\
& + \sum_{\omega,a} \pi(\omega)\delta^{t(\omega)} Ik_a(ik^{\omega}_{a})
\end{aligned}
$$

such that:

$$
\begin{aligned}
&\forall \omega,\ m,\ a, && \sum_{p} fp^{\omega}_{m,p,a} + \sum_{i} fi^{\omega}_{m,i,a} - \left(Tk_a + \sum_{\omega'<\omega} ik^{\omega'}_{a}\right) && \leq 0 && (\tau^{\omega}_{m,a}) && \text{(5a)} \\
&\forall \omega,\ a, && ik^{\omega}_{a} - La_a \left(Tk_a + La_a \sum_{\omega'<\omega} ik^{\omega'}_{a}\right) && \leq 0 && (\iota a^{\omega}_{a}) && \text{(5b)} \\
&\forall \omega,\ m,\ p,\ n, && \sum_{a} M6_{an} fp^{\omega}_{m,p,a}(1 - loss_a) - \sum_{a} M5_{an} fp^{\omega}_{m,p,a} && && && \\
& && + \sum_{f} q^{\omega}_{mpf} M1_{fn} - \sum_{d}\sum_{f} x^{\omega}_{mfpd} M3_{dn} && && && \\
& && - \sum_{i}\sum_{f} zp^{\omega}_{mfpi} M2_{in} && = 0 && (\alpha p^{\omega}_{m,p,n}) && \text{(5c)} \\
&\forall \omega,\ m,\ i,\ n, && \sum_{a} M6_{an} fi^{\omega}_{m,i,a}(1 - loss_a) - \sum_{a} M5_{an} fi^{\omega}_{m,i,a} && && && \\
& && - \sum_{d} y^{\omega}_{mfid} M3_{dn} + \sum_{p} zi^{\omega}_{mpi} M2_{in} && && && \\
& && - (-1)^m \sum_{s} M4_{sn} \left(\sum_{i} ij^{\omega}_{is}\right) && = 0 && (\alpha i^{\omega}_{m,p,n}) && \text{(5d)} \\
&\forall \omega,\ m,\ a,\ p,\ i, && fp^{\omega}_{m,p,a},\ fi^{\omega}_{m,i,a},\ ik^{\omega}_{a} && \geq 0 && &&
\end{aligned}
$$

The objective function contains both the transport/congestion and invesment costs. The congestion cost through arc a, $\tau^{\omega}_{m,a}$, is the dual variable associated with the constraint (5a). This constraint concerns the physical seasonal capacity of arc a, including the possible node-dependent investments.

The constraint (5b) bounds the capacity expansion of each arc a: each year, the investment decided to increase the transport capacity is less than $100 \times La_a$ percent the installed capacity at that year.

The other constraints are market-clearing conditions at each node, depending on whether this node is a production field, an independent trader location, a demand market or a storage site, and depending on whether the transportation costs are supported by the producers or the independent traders.

We consider both pipeline and LNG routes for transport. The liquefaction and regasification costs are included in the transportation costs on the LNG arcs. We assume, in the representation that the physical losses occur at the end nodes of the arcs.

The storage operator optimization (cost minimization) program is given below. The corresponding decision variable is is_s^ω.

Min

$$\sum_{\omega,s} \pi(\omega)\delta^{t(\omega)} Is_s(is_s^\omega) + \sum_{i,\omega,s} \pi(\omega)\delta^{t(\omega)} \left((Ic_s + Wc_s)in_{is}^\omega + Rc_s r_{is}^\omega\right)$$

such that:

$$\forall \omega,\ s, \quad \sum_i r_{is}^\omega - Ks_s - \sum_{\omega' < \omega} is_s^{\omega'} \leq 0 \quad (\beta s_s^\omega) \tag{6a}$$

$$\forall \omega,\ s, \quad is_s^\omega - Ls_s Ks_s - Ls_s \sum_{\omega' < \omega} is_s^{\omega'} \leq 0 \quad (\iota s_s^\omega) \tag{6b}$$

$$\forall \omega,\ s, \quad is_s^\omega \geq 0$$

The storage operator only controls the different investments that dynamically increase the storage capacity of each storage node. The incentive this player has to invest is due to the constraint he must satisfy: the capacity available at each storage site must be sufficient to meet the volumes the independent traders have to store each year in the off-peak season. Capacity expansion is bounded and we used the value $Ls_s = 0.2$.

If we take a closer look at the optimization program of a producer, we will notice that his feasibility set depends on the decision variables of the independent traders. Also, the feasibility set of any independent trader's optimization program depends on the producers' decision variables. The situation is similar for the pipeline and storage operators. This particularity makes our formulation (the KKT conditions) a **Generalized Nash-Cournot problem**. Similarly, the Generalized Nash-Cournot problem can also be formulated as a Quasi Variational Inequality problem (QVI). When the KKT conditions are written, we obtain the Mixed Complementarity Problem.

3 Lemmas, theorems and propositions

Proof of Lemma 1

Theorem 1's proof is straightforward: using equation (1), we can deduce that:

$$\forall b,\ p_{b+1} = p_b^{1+\zeta_b} \tag{7}$$

Hence

$$\begin{aligned} p_{b+\kappa} &= p_{b+\kappa-1}^{1+\zeta_{b+\kappa-1}} \\ &= p_{b+\kappa-2}^{(1+\zeta_{b+\kappa-2})(1+\zeta_{b+\kappa-1})} \\ &= p_{b+\kappa-3}^{(1+\zeta_{b+\kappa-3})(1+\zeta_{b+\kappa-2})(1+\zeta_{b+\kappa-1})} \\ &= \ldots \\ &= p_b^{\Pi_{i=0}^{\kappa-1}(1+\zeta_{b+i})} \end{aligned}$$

Proof of Theorem 1

If we develop $\Pi_{i=0}^{\kappa-1}(1+\zeta_{b+i})$, we find:

$$\Pi_{i=0}^{\kappa-1}(1+\zeta_{b+i}) = \sum_{j=0}^{\kappa} \sum_{\substack{(k_1,k_2,\ldots,k_j) \in \{b,\ldots,b+\kappa-1\} \\ k_1 < k_2 < \ldots < k_j}} \zeta_{k_1}\zeta_{k_2}\ldots\zeta_{k_j} \tag{8}$$

In the sum, the term that corresponds to $j = 0$ is 1 and to $j = 1$ is $\left(\sum_{i=0}^{\kappa-1} \zeta_{b+i}\right)$. Therefore, we can write:

$$\sum_{i=0}^{\kappa-1}(1+\zeta_{b+i}) = \left(1+\sum_{i=0}^{\kappa-1}\zeta_{b+i}\right) + \sum_{j=2}^{\kappa} \sum_{\substack{(k_1,k_2,...,k_j)\in\{b,...,b+\kappa-1\} \\ k_1<k_2<...<k_j}} \zeta_{k_1}\zeta_{k_2}...\zeta_{k_j} \tag{9}$$

Therefore, we have:

$$|\epsilon| = \left|\sum_{j=2}^{\kappa} \sum_{\substack{(k_1,k_2,...,k_j)\in\{b,...,b+\kappa-1\} \\ k_1<k_2<...<k_j}} \zeta_{k_1}\zeta_{k_2}...\zeta_{k_j}\right| \tag{10}$$

and we can write :

$$\begin{aligned}
|\epsilon| &\leq \sum_{j=2}^{\kappa} \sum_{\substack{(k_1,k_2,...,k_j)\in\{b,...,b+\kappa-1\} \\ k_1<k_2<...<k_j}} |\zeta_{k_1}||\zeta_{k_2}|...|\zeta_{k_j}| \\
&\leq \sum_{j=2}^{\kappa} \sum_{\substack{(k_1,k_2,...,k_j)\in\{b,...,b+\kappa-1\} \\ k_1<k_2<...<k_j}} \zeta_{max}^{j} \\
&= \sum_{j=2}^{\kappa} \binom{\kappa}{j} \zeta_{max}^{j} \\
&= (1+\zeta_{max})^{\kappa} - 1 - \kappa\zeta_{max}
\end{aligned}$$

The last equality is obtained exploiting the Newton binomial theorem.

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